A

TREATISE

OF SUCH

Mathematical Instruments,

As are usually put into a

PORTABLE CASE:

CONTAINING their various Uses in

ARITHMETIC,	ARCHITECTURE,
GEOMETRY,	SURVEYING,
TRIGONOMETRY,	&c. &c.

DESIGNED

For the Benefit of ENGINEERS, ARCHITECTS, SURVEYORS, and for Young Students in the MATHEMATICS.

To which is prefixed,

A Short ACCOUNT of the Authors who have treated on the PROPORTIONAL COMPASSES and SECTOR.

By J. ROBERTSON, *F.R.S.*

LONDON:

Printed for T. HEATH, *Mathematical Instrument-maker,* opposite *Exeter-Change,* in the *Strand*; J. HODGES, at the *Looking-Glass* on *London-Bridge*; and J. FULLER, at the *Bible and Dove* in *Ave-Mary-Lane.*

M. DCC. XLVII.

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M.DCC.XLVII.

TO

Martin Folkes, *Eſq*;

PRESIDENT

OF THE

Royal-Society.

SIR,

THAT tribute, which is juſtly due from the lovers of ſcience, to thoſe who are the moſt competent judges thereof, I hope will excuſe my preſumption, in addreſſing this work to a gentleman ſo

deſervedly diſtinguiſhed for his univerſal knowledge.

I cannot be ſo vain to imagine, that any thing contained in the following ſheets, will either appear new to you, or in a better light than that in which you have already conceived it: This work pretends no farther, than to render the uſe of ſeveral mathematical inſtruments familiar to young ſtudents, and all who have occaſion for their aſſiſtance; a deſign which I flatter myſelf you will approve.

I am, with all poſſible reſpect,

SIR,

Your moſt humble, and

moſt obedient ſervant,

John Robertſon.

TO THE

READER.

IT is needless to enumerate the many purposes, to which *mathematical instruments* serve; their use seems quite necessary to *persons* employed in most of the active stations in life.

The *Architect*, whether *civil*, *military*, or *naval*, never offers to effect any undertaking, before he has first made use of his *rule* and *compasses*; and fix'd upon a scheme or drawing, which unavoidable requires those *instruments*, and others equally necessary.

The *Engineer*, cannot well attempt to put in execution any design, whether for *defence*, *offence*, *ornament*, *pleasure*, &c. without first laying before his view, the plan of the whole; which is not to be conveniently performed, but by *rulers*, *compasses*, &c.

 There

There are indeed, very few, if any good *Artificers*, who have not in some measure, occasion for the use of one or more *mathematical instruments*; and whenever there is required, an accurate drawing of a thing to be executed, or represented; that collection of instruments, usually put in *portable cases*, is then absolutely necessary: And of these, the most common ones, or others applicable to like service, must have been in use, ever since mankind have had occasion to provide for the necessary conveniencies of life: But the *parallel ruler*, the *proportional compasses*, and the *sector*, are not of any great antiquity.

However, by means of the opportunity, which the author had of consulting most, if not all, that have wrote on this subject *; he thinks it will sufficiently appear from what follows, who were the inventors of these latter instruments; and when they were first known and made use of.

I. *Gaspar Mordente*, in his book *on the compasses*, printed in folio at *Antwerp*, 1584; gives the construction and use of an instrument, invented by his brother *Fabricius Mordente*, in 1554. The instrument consists of two flat legs, moveable

* In the collection of *William Jones*, Esq;

able round a joint like a common pair of compaſſes; but the points are turn'd down at right angles to the legs, ſo as to make but one point when the legs are cloſe. In each leg there is a groove, with a ſlider fitted to it, carrying a perpendicular point; ſo that theſe alſo appear like one point when the legs are cloſe, and the ſliders are oppoſite. This compaſs is jointly uſed with a rod, containing a ſcale of equal parts; 30 of which, are equal to the length of each leg. As the operations with this compaſs, depend on the properties of ſimilar triangles, therefore its principles are the ſame with thoſe of the ſector: And moſt, or all the problems that are perform'd by the line of lines only, can with almoſt the ſame eaſe, be performed by theſe; the tranſition from this inſtrument to the ſector is very natural and eaſy.

The uſe of this inſtrument, is exemplified in problems concerning lines, ſuperficies, ſolids, and meaſuring of inacceſſible diſtances.

The author, page 22, ſays, he invented an inſtrument there deſcribed; which is our parallel ruler with parallel bars: The parallel ruler with croſs bars, is a more modern contrivance.

II. *Daniel Speckle*, in the year 1589, publiſhed in folio, his *military archi-*

tecture, at *Strasburg*; where he was architect. In his second chapter, he takes notice of compasses then in use of a curious invention, whose center could be moved forwards or backwards, so that by the figures and divisions mark'd theron, a right line could be readily and correctly divided into any number of equal parts, not exceeding 20. This instrument has been since called the *proportional compasses*.

In the same chapter he mentions another compasses, with an immoveable center, and broad legs, whereon were drawn lines proceeding from the center, and divided into equal parts; whereby a right line could be divided into equal parts not exceeding 20; because the divisions on the lines still kept the same proportion to whatever distance the legs were opened. This instrument was afterwards call'd the *sector*.

III. Dr. *Thomas Hood*, printed at *London*, *Anno* 1598, a quarto book, intituled, *The making and use of a Geometrical Instrument called a Sector*. This instrument consists of two flat legs, moveable about a joint; on these are sectoral lines, of equal parts, of polygons, and of superficies; that is, lines so disposed, as to make all the operations that depend on similar triangles quite easy,

and

and that without the laying down of any figure. To the legs is fitted a circular arc, an index moveable on a joint, and ſights, whereby it is made fit to take angles.

IV. *Chriſtopher Clavius*, in his *practical geometry*, printed in quarto at *Rome*, *an*. 1604, in page 4, ſhews the conſtruction and uſe of an inſtrument, which he calls the *inſtrument of parts*; it conſiſts of two flat rulers moveable on a joint; on one ſide of theſe legs, are the ſectoral lines of equal parts; on the other ſide, are thoſe of the chords: After ſhewing ſome of their uſes, he concludes with ſaying, he is ſenſible of many others to which it may be applied, but leaves them for the exerciſe of the reader to diſcover.

V. *Levinus Hulſius*, in his book of *mechanical inſtruments*, printed in quarto at *Frankfort*, *An*. 1605; gives, in the third part, the deſcription and uſe of an inſtrument, which *Juſtus Burgius* call'd the *proportional compaſs*. *Hulſius* ſays, the uſe of it had not been publiſhed before, although the inſtrument had been long known.

VI. *Anno* 1605, *Philip Horſcher*, M. D. publiſhed at *Mentz*, a quarto book, containing the uſe and conſtruction of the *proportional compaſſes*. This

author does not pretend to be the inventor; but that ſeeing ſuch an inſtrument, he thought he could, from *Euclid*, ſhew its conſtruction and the grounds of its operations.

VII. *Anno* 1606, *Galilæus* publiſhed in *Italian*, a treatiſe of the uſe of an inſtrument which he calls, *The geometrical and military compaſs.* On this inſtrument are deſcribed ſectoral lines of equal parts, ſurfaces, ſolids, metals, inſcribed polygons, polygons of given areas, and ſegments of circles. In the preface to an edition of this book, printed at *Padua*, *Anno* 1640. By *Paola Frambotti*, *Galilæus* ſays, that on account of the opportunity he had of teaching mathematics at *Padua*, he thought it proper to ſeek out a method of ſhortening thoſe ſtudies. In another part of the preface he ſays, that he ſhould not have publiſhed this tract, but in vindication of his own reputation; for he was informed that a perſon had by ſome means or other, got one of his inſtruments, and pretended to be the inventor, although himſelf had taught it ever ſince the year 1597.

VIII. *Anno* 1607, *Baldeſſar Capra*, publiſhed a treatiſe of the conſtruction and uſe of the *compaſs* of *proportion*, (or ſector.) He claims the invention

vention of this inſtrument; and hence aroſe a diſpute between *Galilæus* and *Capra*; ſome particulars of which has been mentioned by ſeveral, and particularly by *Thomas Salusbury*, Eſq; in his life of *Galilæus*, publiſhed at the end of the ſecond volume of his *mathematical collections and tranſlations*, at in fol. *London*, *Anno* 1664.

IX. *An.* 1610, *John Remmelin*, M. D. publiſhed at *Frankfort*, a quarto edition of two tracts of *John Faulhaber*; one of theſe contains the uſe of the *ſector*, on which are lines of equal parts, ſuperficies, ſolids, metals, chords, &c. He ſays, that *G. Brendel*, a painter, uſed this inſtrument in perſpective painting.

X. *D. Henrion*, in his *mathematical memoirs*, *Anno* 1612, gave a ſhort tract of the uſe of the compaſs of *proportion* (or *ſector.)* In 1616 he printed a book of the uſe of the ſector; and a fifth edition, in the year 1637, the preface to which, ſeems to be wrote in the year 1626, wherein he ſays, that about the year 1608, he had ſeen in the hands of *M. Alleaume*, engineer to the king of *France*, one of theſe ſectors; whereupon he wrote ſome uſes of it, which he publiſhed in his memoirs, as above. He alſo declares, that before his firſt

publication, he had not ſeen any book on the uſe of the ſector, and therefore calls what he publiſhes his own. He charges Mr. *Gunter* with having uſed many of his propoſitions. This author printed at *Paris* 1626, an octavo book of logarithms, at the end of which is a tract call'd logocanon, or the proportional ruler; which is a deſcription and uſe of an inſtrument, he calls a lattice, (perhaps from the chequer-work made by lines drawn thereon) which operates the problems performed by the *french* ſectors very accurately.

XI. *Anno* 1615, *Stephen Michaelſpackers*, publiſhed in quarto at *Ulm*, a treatiſe of the *proportional rule and compaſs of* G. *Galgemeyer*, reviſed by G. *Brendel*, a painter at *Laugingen*. On theſe proportional compaſſes, are lines of equal parts, of polygons, ſuperficies, ſolids, ratio of the diameter to the circumference; reduction of planes, and reduction of ſolids. The uſe and conſtruction of theſe lines, are ſhewn by a great variety of examples.

XII. *Benjamin Bramer*, in his book of *the deſcription of the proportional ruler and parallelogram*, printed in quarto at *Marpurg*, *anno* 1617; ſays, his ruler is applicable to the ſame uſes as *Juſtus Burgius*'s inſtrument. *Bramer*'s inſtrument

ment confifts of a ruler, on which are lines of equal parts, of fuperficies, of folids, of regular folids, of circles, of chords, and of equal polygons; at the beginning of each fcale, is a pin-hole, whereby he can apply the edge of another ruler, and fo conftitute a fector for each fcale.

XIII. *Anno* 1623, *Adriano Metio Alcmariano*, printed at *Amfterdam* a quarto book, fhewing the ufe of an inftrument called the *rule of proportion.* In his dedication, he fays, that whilft he was reveiwing fome things relating to practical geometry, he met with *Galiléo*'s book of the ufe of the fector, which gave him opportunity to improve on it, and occafioned the publifhing of this book.

XIV. Mr. *Edmund Gunter*, profeffor of aftronomy in *Grefham* colledge, printed at *London*, *an.* 1624, a quarto book, called *the defcription and ufe of the fector*; on which are fectoral lines, 1ft. of equal parts; 2d. fuperficies; 3d. folids; 4th. fines and chords; 5th. tangents; 6th. rhumbs; 7th. fecants: Alfo lateral lines of, 8th. quadratures; 9th. fegments; 10th. infcribed bodies; 11th. equated bodies; 12th. metals: On the edges are a line of inches and a line of tangents.

Mr.

Mr. *Gunter* does not ſay any thing concerning the invention, and has no preface; but at the end of the tract, in a concluſion to the reader, he ſays, that the ſector was thus contrived, moſt part of the book wrote, and many copies diſperſed, more than ſixteen years before, *&c.* this article being wrote *May* 1, 1623, brings the time he ſpeaks of to about the year 1607, which was before the time *Henrion* ſays he firſt ſaw the ſector.

The ſcales of logarithm numbers, ſines, and tangents, were firſt publiſhed in 1624, in *Gunter*'s deſcription of the croſs ſtaff.

XV. *Mutio Oddi* printed at *Milan*, *an.* 1633, a quarto book, called *the conſtruction and uſe of the compaſſo polimetro*, (or ſector.) The lines on this inſtrument, were ſuch as were common at that time.

In the preface he ſays, that about the year 1568, *Commandine*, who then taught at *Urbino*, did contrive an inſtrument to divide lines into equal parts, which was done at the requeſt of a gentleman named *Bartholomew Euſtachio*, who had frequent occaſion for the diviſion of right lines.

He farther ſays, that about that time, *Guidobaldo*, marqueſs of *Monte*, who lived

lived at *Urbino* for the ſake of *Commandine*'s company, being frequently at the houſe of *Simone Boraccio*, who made *Commandine*'s proportional compaſſes, did contrive, and cauſe to be made, an inſtrument with flat legs, (like the ſector) which performed the operations of the compaſs more eaſily. *Oddi* ſays alſo, that great numbers were made, and in few years, had many uſeful and curious additions, with treatiſes wrote on its uſe in diverſe languages, and called by different names, which occaſioned the doubt of whom was the true author, every one having found means to ſupport his cauſe: But *Oddi* ſays, he not intending to decide the diſpute, leaves it to time to diſcover; and ſeems contented to have pointed out who was the firſt inventor; his chief intention being that of making the uſe public, and the conſtruction eaſy to workmen.

The following authors have alſo wrote on the ſector, and ſectoral lines.

XVI. *An.* 1634, *P. Petit*, printed in 8vo. at *Paris*, a treatiſe on the ſector. He thinks *Galilæus* was the inventor.

XVII. *An.* 1635, *Mattheus Berneggerus* printed at *Strasburg* a 4to. edition of *Galilæus*'s book on the ſector, which conſiſts of two parts: To this is added a third part, ſhewing the conſtruction of

of *Galilæus*'s lines, and ſome additional uſes and tables.

XVIII. *An.* 1639, *Nicholas Foreſt Ducheſne* printed at *Paris*, in 12mo. a book of the ſector. He ſeems to be little more than a copier of *Henrion.*

XIX. *An.* 1645, *Bettinus* in his *Apiaria univerſa*, &c. *apiar.* 3d. p. 95, and *apiar.* 12, p. 4. In his *Ærariarum philo. math.* 4to. *an.* 1648, vol. I. p. 262. In his *Recreationum math. appiariæ*, &c. 12mo. *an.* 1658, p. 75, applies the ſector to muſic.

XX. *John Chatfield* printed at *London*, in 12mo. his *trigonal ſector*, *anno* 1650.

XXI. *An.* 1656, *Nicholas Goldman* printed at *Leyden*, in folio, his *treatiſe on the ſector*. He ſays that *Galilæus* was the firſt who publiſhed the deſcription of the ſector, an invention uſeful in all parts of the mathematics, and other affairs of life.

XXII. *John Collins* printed at *London*, in 4to. his book of *the ſector on a quadrant*, *an.* 1659.

XXIII. *Pietro Ruggiero*, in his *military architecture*, in 4to. printed at *Milan*, *an.* 1661, p. 230, applys the ſector to the practice of fortification.

XXIV. *An.* 1662, *Gaſpar Schottus* printed at *Strasburgh* his *matheſis cæ-*

ſaræa

ſaræa, in 4to. in which he gives a deſcription and uſe of the ſector: In the preface he mentions *Galilæo* as the inventor of the ſector.

XXV. *J. Templar* printed in 12mo. at *London*, *an.* 1667, a book called *the ſemicircle on a ſector.* He ſays, the applying of Mr. *Forſter*'s line of verſed ſines to the ſector, was firſt publiſhed *an.* 1660, by *John Brown*, mathematical inſtrument-maker in *London.*

XXVI. *Daniel Schwenter* in his *practical geometry*, reviſed and augmented by *George Andrew Becklern*, printed in 4to. at *Nuremberg*, *an.* 1667, treats on the deſcription and uſe of the ſector.

XXVII. *John Caramuel* printed at *Campania*, *an.* 1670, his *matheſis nova*, in 2 vols. folio. In the 2d vol. p. 1158, he treats on the ſector, relates the conteſt between *Galilæus* and *Capra*, and thinks the ſame might have been objected againſt others, as well as againſt *Capra*: He alſo ſays, that *Clavius* had ſuch an inſtrument before that of *Galilæus* appeared; and *Clavius* having taught for a long time at *Rome*, had many ſcholars, ſome of whom might have carried his inſtruments to ſeveral countries. *Caramuel* mentions a ſtory of a *Hollander* ſhewing to *Galilæus* an

inſtrument

inſtrument of this ſort, that he had brought from his country, and of which *Galilæus* took a copy.

XXVII. *John Brown*, in his book on *the triangular quadrant*, printed in 8vo. at *London*, *an.* 1671.

XXIX. *John Chriſtopher Rohlhans*, in his *math. and optical curioſities*, printed in 4to. at *Leipſic*, *an.* 1677, p. 216.

XXX. *An.* 1683, *Staniſlawa Solſkiego* printed at *Kracow*, his *geometria et architectura Polſki*, in folio. p. 69, treats on ſome ſectoral lines.

XXXI. *Henrick Jaſper Nuis*, printed at *Tezwolle*, in 4to. his *Rectangulum catholicum geometrica aſtronomicum*, *an.* 1686.

XXXII. *De Chales*, in his *curſus mathem.* printed at *Leyden*, in 2 vols. fol. *an.* 1690. Vol. 2d. p. 58, relates the conteſt between *Galilæus* and *Capra*, and aſcribes the invention of the proportional compaſs to Dr. *Horcher*, or *Juſtus Burgius*.

XXXIII. *An.* 1691, an edition in 8vo. of Mr. *Ozanam*'s treatiſe of the ſector, was printed at the *Hague*.

XXXIV. *P. Hoſte* printed at *Paris* his courſe of mathematics, in 3 vols. 8vo. *an.* 1692. In vol. 2d. p. 27. he gives a tract on the ſector.

XXXV.

XXXV. *Thomas Allingham* in his *ſhort treatiſe on the ſector*, in 4to. *Lond.* 1698.

XXXVI. *J. Good*, in his *treatiſe on the ſector*, in 12mo. *Lond.* 1713.

XXXVII. *Chriſtian Wolfius*, in his *math. lexicon*, 8vo. printed at *Leipſic*, *an.* 1716, under the word *circinus proportionum*, relates, that *Levinus Hulſius*, in his treatiſe on the proportional compaſſes, printed at *Frankfort* the 10th of *May*, 1603, ſays, that he firſt ſaw the ſaid inſtrument at *Ratisbon*, on the day of the imperial dyet: That he had ſold them far and near before 1603; and that it had been inaccurately copied in ſeveral places: *Wolfius* ſays farther, that *Juſtus Burgius* was certainly the inventor, but uſed to let his inventions lye unpubliſhed.

He then relates the conteſt between *Galilæus* and *Capra*, and ends with ſhewing the difference between the inſtruments of *Burgius* and *Galilæus*.

XXXVIII. *M. Bion*, in his conſtruction of mathematical inſtruments, tranſlated by *Edmund Stone*, fol. *Lond.* 1723.

XXXIX. Mr. *Belidor*, in his *new courſe of math.* in 4to. p. 364, *Paris*, 1725.

XL. *Roger Rea*, in his *sector and plane scale compared*, 8vo. *Lond.* 1727, 2d edition.

XLI. *Vincent Tosco*, in his *compendium of the math.* in 9 vols. 8vo. *Madrid*, 1727, vol. I. p. 359.

XLII. *Jacob Leupold*, in his *theatrum arithmetico-geometricum*, in fol. *Leipsic*, 1727. p. 86, gives a detail of the inventors of the proportional compasses and sector, which goes on to p. 121, and then he gives a list of the authors who have wrote on proportional instruments, *viz. Bramer*, 1617; *Capra*, 1607; *Casati*, 1664; *Conette*, 1626; *Dechales*, 1690; *Dolz*, 1518; *Faulhaber*, 1610; *Galgemeyer*, 1615; *Brendell*, 1611; *Galilæus*, 1612; *Goldman*, 1656; *Horscher*, 1605; *Horen*, 1505; *Hulsius*, 1604; *Clavius*, 1615; *Lockmanns*, 1626; *Metius*, 1623; *Patridge* ---; *de Saxonica*, 1519; *Scheffelts*, 1697; *Steymann*, 1624; *Uttenhoffers*, 1626.

XLIII. *Samuel Cunn*, in his *new treatise on the sector*, 8vo. *London*, 1729.

XLIV. *William Webster*, in his appendix to a translation of *P. Host*'s mathematics, 8vo. 2 vols. *London*, 1730.

There

There may be ſeveral other authors who have wrote on the conſtruction and uſe of the ſector, or on ſome of the ſectoral lines; but thoſe above, are all that have come to hand; and indeed theſe are many more than are wanted to determine this enquiry; which may be collected chiefly, from *Mordente*, *Speckle*, *Hood*, *Clavius*, *Hulſius*, *Galilæus*, *Oddi*, *Saluſbury*, *Caramuel*, *Dechales*, *Wolfius*, and *Leupold*; the others ſerving only to inform the reader what works are extant on this ſubject. From the whole he may obſerve, that there are few countries in *Europe*, but have one or more treatiſes on the proportional compaſſes and ſector, in their own language; and this is ſufficient to ſhew, that theſe inſtruments have been in univerſal eſteem.

What is done in this eſſay, and in the following work, is ſubmitted to the reader's judgment; who, it is hoped, will excuſe ſuch little faults, or inaccuracies, as may have eſcaped the author's notice; his intention being no more, than that of giving aſſiſtance to beginners in the mathematical ſtudies.

October, 27,
1746.

There may be several good authors who have wrote on the construction and use of the sector, as [illegible] the national [illegible] for these [illegible] as well [illegible] determine this enquiry [illegible] Abel Chr[illegible], Alvary, [illegible] Salisbury, Gerhard, Beling, [illegible] and Lewis; [illegible] only to inform the reader, that [illegible] are extant on this subject. From the whole he may observe, that very few countries in Europe [illegible] have one or more treatises on the geometrical compasses and sector, in their own language; and this is sufficient to shew, that their instruments have been in universal esteem.

What is done in this essay, and in the following work, is submitted to the reader's judgment; and he, unbiassed, will excuse some faults or inaccuracies, as they have escaped the author's notice; [illegible] ing no more, than that of [illegible] assistance to beginners in the mathematical studies.

October 17, [illegible]

E H. E.

THE CONTENTS.

 2d.

The figures referred to, are contained in seven copper plates.

ERRATA.

ERRATA.

Page 8, line laſt, read, on grooves. P. 9, l. 17, r. than. P. 10, l. 13, r. ABCD. P. 17, l. 33, r. arcs. P. 22, l. 16, r. $13\frac{1}{2}$, $16\frac{1}{2}$. P. 24, l. 13, r. F. P. 28, l. 25, dele that, and r. and $\angle$ BAC ſhall. P. 94, l. 4, r. $\angle$ D. P. 95, l. 20, r. da. P. 96, l. 27, r. AB.

THE

DESCRIPTION *and* USE

OF A

CASE,

OR

PORTABLE COLLECTION,

Of the moſt Neceſſary

Mathematical Inſtruments.

SECT. I.

CASES of *Mathematical Inſtruments* are of various ſorts and ſizes; and are commonly adapted to the fancy or occaſion of the perſons who buy them.

THE ſmalleſt collection in a caſe, commonly conſiſts of,

I. A *pair of compaſſes*, one of whoſe points may be taken off, and its place ſupplied with,

A *crayon* for lead or chalks.

A *drawing pen* for ink.

II. A *plane ſcale.*

WITH theſe inſtruments only, a tolerable ſhift may be made to draw moſt mathematical figures.

BUT in ſets, called *complete pocket-caſes*, beſide the inſtruments above, are the following.

III. A ſmaller pair of *compaſſes.*

IV. A pair of *bows.*

V. A *black-lead pencil*, with a *cap* and *feeder.*

VI. A *drawing-pen* with a *protracting-pin.*

VII. A *protractor.*

VIII. A *parallel-ruler.*

IX. A *ſector.*

IN the beſt ſort of caſes, the plane ſcale, protractor, and parallel ruler, are included in one inſtrument.

THE common, and moſt eſteemed ſize of theſe inſtruments, is ſix inches; though they are ſometimes made of other ſizes, and particularly of four inches and a half.

Note, The ſize of a caſe is named from the length of the ſcale or ſector.

A more uſeful caſe of inſtruments, is the *box-caſe*; whoſe top within, contains the rulers and ſcales: The compaſſes, drawing-pen, *&c.* lie in the partitions of a drawer, that drops into the bottom part of the caſe, but not quite to the bottom; leaving room under it for *black-lead pencils*, *hair pencils*, *Indian ink*, *colour cells*, &c. and beſide the inſtruments already

already enumerated, in this ſort of caſe are put

X. A *tracing-point.*

XI. A pair of *proportional compaſſes.*

XII. A *gunner's callipers.*

But the caſe of inſtruments called the *magazine*, is the moſt complete collection; for this contains whatever can be of uſe in the practice of *drawing*, *deſigning*, &c. and as the greateſt part of theſe inſtruments are ſcarcely ever uſed but in the ſtudies or chambers of thoſe who have occaſion for them; therefore it will be uſeleſs to inſiſt on pocket caſes; for few perſons care to load themſelves with the carriage of what is called a *complete ſet*, unleſs it be children who are learning the ſcience, or elſe, pretenders to art.

Sect. II.

Of the Compasses.

Those Compaſſes are reckoned the beſt, part of whoſe joint is ſteel; and where the *pin* or *axle* on which the joint turns, is a ſteel ſcrew; for the oppoſition of the metals makes them wear more equable: and by means of the ſcrew axle, with the help of a *turn-ſcrew*, (which ſhould have a place in the caſe) the compaſſes can be made to move in the joint, ſtiffer or eaſier, at pleaſure. If this motion is not uniformly ſmooth, it renders the inſtrument leſs accurate in uſe. Their points ſhould be of ſteel, and

Do you mean on the bows?

pretty well hardened, elſe in taking meaſures off the ſcales, they will bend, or be ſoon blunted. They alſo ſhould be well poliſhed, whereby they will be preſerved free from ruſt a long time.

To one point of the ſmaller compaſſes, it is common to fix in the ſhank a ſpring, which by means of a ſcrew, moves the point; ſo that when the compaſs is opened nearly to a required diſtance, by the help of the ſcrew the points may be ſet exactly to that diſtance; which cannot be done ſo well by the motion in the joint.

THE uſe of theſe leſſer compaſſes, is to transfer the meaſures of diſtances from one place to another; or, to deſcribe obſcure arcs.

OF the large ſized compaſſes, thoſe are eſteemed the beſt, whoſe moveable points are locked in by a ſpring and catch fixed in the ſhank; for if this ſpring be well effected, the point is thereby kept tight and ſteady; the contrary of which frequently happens, when the point is kept in by a ſcrew in the ſhank.

THE uſe of theſe compaſſes are to deſcribe arcs or circumferences with given radius's: and it is eaſy to conceive, that theſe arcs or circumferences can be deſcribed, either obſcurely by the ſteel point; in ink, by the ink point; in black lead or chalks, by the crayon; and with dotts, by the dotting-wheel; for either of them may be fixed in the ſhank in the place of the ſteel point.

As

As the dotting-wheel has not hitherto been effected, ſo as to deſcribe dotted lines or arcs, with any tollerable degree of accuracy, it ſeems therefore to be uſeleſs: and, indeed, dotted lines of any kind are much better made by the drawing-pen.

The drawing-pen point, and crayon, have generally (in the beſt ſort of caſes) a ſocket fitted to them: ſo that they occupy but one of the holes, or partitions, in the caſe.

The ink, and crayon points, have a joint in them, juſt under that part which locks into the ſhank of the compaſſes; becauſe the part below the joint ſhould ſtand perpendicular to the plane on which the lines are deſcribed, when the compaſs is opened.

If inſtead of the larger compaſs being made with ſhifting points, there were two pair put into the caſe; to one of which the ink point was fixed, and to the other the crayon point; this would ſave the trouble of changing the points in the compaſs at every time they were uſed; and would increaſe the expence, or bulk of the caſe, but a trifle.

Sect. III.

Of the Bows.

The bows are a ſmall ſort of compaſſes, that commonly ſhut into a hoop, which ſerves as a handle to them. Their uſe is to deſcribe arcs, or the circumferences of circles,

whoſe radius's are very ſmall, and could not be done near ſo well by larger compaſſes.

Sect. IV.

Of the Black-lead Pencil, Feeder, and Tracing Point.

THE *Black-lead pencil* is uſeful to deſcribe the firſt draught of a drawing, before it is marked with ink; becauſe any falſe ſtrokes, or ſuperfluous lines, may be rubb'd out with a handkerchief or piece of bread.

The *Feeder* is a thin flat piece of metal; and ſerves either to put ink between the blades of the drawing-pen, or to paſs it between the points, when the ink by drying, does not flow freely.

The *Tracing Point* is a pointed piece of ſteel, and is commonly at the other end of the handle to which the *feeder* is fix'd. Its uſe, is to help the making of an exact copy of a drawing or print, which may be done as follows.

On a piece of paper, large enough to cover the thing to be copied, let there be ſtrewn the ſcrapings of *red chalk*, or of *black chalk*, or of *black lead*; rub theſe on the paper, ſo that it be uniformly covered; and wipe off, with a piece of muſlin, as much as will come away with gentle rubbing. Lay the coloured ſide of this paper, next to the vellum, paper, *&c.* on which the drawing is to be made: on the back of the colour'd paper, lay the drawing, *&c.* to be copied. Secure all the corners

 with

with weights, or pins, that the papers may not ſlip: trace the lines of the thing to be copied, with the *tracing point*; and the lines ſo traced will be impreſs'd on the clean paper.

And thus, with care, may a drawing or print, be copied without being much damaged.

Note, The coloured paper will ſerve a great many times.

Sect. V.

Of the Drawing-Pen and Protracting-Pin.

THE *Drawing-pen* is an inſtrument uſed only for drawing of right lines; and conſiſts of two blades, with ſteel points, fix'd to a handle. The blades by being a little bent, cauſe the ſteel points to come nearly together; but by means of a ſcrew paſſing through both of them, they are brought cloſer at pleaſure, as the line to be drawn ſhould be ſtronger or finer.

In uſing this inſtrument, put the ink between the blades with a common *pen*, or with the *feeder*; and (by the ſcrew) bring them to a proper diſtance for drawing the intended line: hold the *pen* a little inclined, and ſo that both blades touch the paper; then may a line be drawn very ſmooth, and of equal breadth, which could not be done ſo well with a common *pen*.

Note, Before the *drawing-pen* is put into the case, the ink should be wiped from between the blades; otherwise they will soon rust and spoil, especially with common ink. And that they may be clean'd easily, one of the blades should move on a joint.

The directions given about this *drawing-pen*, will serve for the *drawing-pen* point, used with the compasses.

The *Protracting pin* is a piece of pointed steel (like the point of a needle) fixed into one end of a part of the handle of the *drawing-pen*; into which, the piece with the *pin* in it, generally screws. Its use is to point out the intersections of lines; and to mark off the divisions of the protracter, as hereafter directed.

Sect. VI.

Of the Parallel-Ruler.

THIS instrument consists of two *Rulers*, connected together by two metal bars, moving easily round the rivets which fasten their ends; these bars are so placed that both have the same inclination to each *Ruler*; whereby they will be *Parallel* at every distance, to which the bars will suffer them to receed.

But the best *Parallel-Rulers* are those, whose bars cross each other, and turn on a joint at their intersection; one end of each bar moving on a centre, and the other ends sliding on grooves as the *Rulers* receed.

This

those end, which lie on the same ruler

THIS inſtrument is very uſeful in delineating civil and military architecture, where there are many *Parallel* lines to be drawn; and alſo in the ſolution of ſeveral geometrical *Problems*; ſome of which are as follows.

PROBLEM I.

A right line A B *being given, to draw a line parallel thereto, that ſhall paſs thro' a given point* C (Fig. 1.)

CON. Apply one edge of the *parallel-ruler* to the given line A B; preſs that *ruler* tight againſt the paper with one hand, move the other untill its edge cut the point C; there ſtay the *ruler*, and by its edge draw a line thro' C, then this line will be *parallel* to A B.

IF the point C happens to be farther from the line A B, then the *rulers* will open to; ſtay that *ruler* neareſt to C, and bring the other cloſe to it, where let it reſt, and move forward the *ruler* neareſt to C, and ſo continue till one *ruler* is brought to the point intended.

THE manner of uſing the *parallel-ruler* as here directed, is underſtood to be the ſame in the ſolution of the following PROBLEMS.

PROBLEM II.

A right line A B *being given, to divide it into any propos'd number of equal parts; ſuppoſe* 5. (Fig. 2.)

CON. Draw the indefinite right line B C, ſo as to make with A B, any angle at pleaſure: with

with any convenient opening of the compasses, lay off on B C, the required number of equal parts, *viz.* 1, 2, 3, 4, 5; lay the edge of the *parallel-ruler* by the points 5 and A, and *parallel* thereto, thro' the points 4, 3, 2, 1, draw lines; then A B, by the intersection of those lines will be divided into 5 equal parts.

PROBLEM III.

Any right lined polygon being given, to make a right lin'd triangle of equal area.

EXAM. I. *To make a triangle of equal area to the quadrilateral* A B C D. (Fig. 3.)

CON. Prolong A B; ~~draw~~ C B; and thro' D, draw D E *parallel* to C B, cutting A E in E; then a line drawn from C to E forms the triangle A C E, of equal area to A B D C.

CB

EXAM. II. *Given the pentagon* A B C D E; *requir'd to make a triangle of equal area.* (Fig 4.)

CON. Produce D C to F; draw A C; thro' B, draw B F *parallel* to A C; and draw A F. Then the area of the trapezium A F D E will be equal to the area of the *pentagon* A B C D E.

Again. Produce E D towards G; draw A D; thro' F, draw F G *parallel* to A D, and draw A G. Then the area of the triangle A G E, will be equal to that of the trapezium A F D E; and consequently, to that of the *pentagon* A B C D E.

D GE

EXAM. III. *To make a triangle equal in area to the Hexagon*, A B C D E F. (Fig. 5.)

CON. Draw F D, and *parallel* thereto, thro'

thro' E, draw E G. Produce C D to G, and draw G F. Then the triangle F G D is equal to the triangle F E D, and the given *Hexagon* is reduced to the *Pentagon* A B C G F equal in area.

Again. Draw A G; thro' F, draw F H *parallel* to A G. Produce C G to H; draw A H, and the *pentagon* is reduced to the trapezium A B C H.

Lastly, Draw A C, and *parallel* thereto, thro' H, draw H I. Produce B C to I, and draw A I. Then the trepezium is reduced to the triangle A B I, which is equal in area to the given *Hexagon* A B C D E F.

Exam. IV. *Given the nine sided figure* A B C D E F G H I, *to make a triangle of equal area.* (Fig. 6.)

Con. 1st, Draw I B, and thro' A draw A, K *parallel* to I B. Produce H I to K, and draw B K; so the three sides H I, I A, A B, are reduced to the two sides H K, K B.

2d, Draw K C, and thro' B draw B L *parallel* to K C; and draw K L, and the three sides D C, C B, B K, are reduced to the two sides D L, L K.

3d, Draw K G; thro' H, draw H M, *parallel* to K G, and draw K M; so the three sides K H, H G, G F, are reduced to the sides K M, and M F.

4th, Draw K F; thro' M, draw M N, *parallel* to K F, and draw K N; so the three sides K M, M F, F E, are reduced to two sides K N, N E.

5th, Draw L N, and thro' K, draw K O, *parallel* to L N. Produce E N to O, and draw

draw L O; ſo the three ſides E N, N K, K L, are reduced to the two ſides E O, O L.

Laſtly, Draw L E, and thro' D, draw D P *parallel* to L E. Produce O E to P, and draw L P; ſo ſhall the triangle O L P be equal in area to the given nine ſided figure.

PROCEEDING in the ſame manner; a figure of any number of ſides may be reduced to a triangle of equal area.

SECT. VII.

Of the PROTRACTOR.

THE *Protractor*, is an inſtrument of a ſemicircular form; being terminated by a right line repreſenting the diameter of a circle, and a curve line of half the circumference of the ſame circle. As at Fig. 7. The point C, (the middle of A B) being the centre of the ſemicircumference A D B, which ſemicircumference is divided into 180 equal parts call'd degrees; and for the convenience of reckoning both ways, is numbered from the left hand towards the right, and from the right hand towards the left, with 10, 20, 30, 40, &c. to 180, being the half of 360, the degrees in a whole circumference. The uſe of this inſtrument is to *protract*, or lay down an angle of any number of degrees, and to find the number of degrees contain'd in any given angle.

BUT this inſtrument is made much more commodious, by transferring the diviſions on the ſemicircumference to the edge of a *ruler*, whoſe

whose side E F is *parallel* to A B; (see Fig. 7.) which is done by laying a *ruler* on the centre C, and the several divisions on the semicircumference A D B, and marking the intersections of that *ruler* on the line E F, which may easily be conceiv'd by observing the lines drawn from the centre C to the divisions 90, 60, 30; so that a *ruler* with these divisions mark'd on 3 of its sides and numbered both ways, as in the *Protractor*, (the fourth or blank side representing the diameter of the circle) is of the same use as a *Protractor*, and is much better adapted to a case.

THAT side of the instrument on which the divisions are mark'd, is call'd the graduated side, or limb of the instrument, which should be sloped away to an edge, whereby the divisions on the limb will be much easier pointed off.

PROBLEM. IV.

A number of degrees being given; to protract, or lay down an angle whose measure shall be equal thereto. And an angle being protracted, or laid down, to find what number of degrees measures that angle.

EXAM. I. *To draw a line from the point* A, *that shall make an angle with the line* A B *of* 48 *deg*. Fig. 8.

APPLY the blank edge of the protractor to the line A B, so that the middle or centre thereof (which is always mark'd) may fall on the point A; then with the protracting-pin, make a mark on the paper against the division on

on the limb of the inſtrument numbered with the degrees given; (*viz.* 48.) counting from the right hand towards the left; a line drawn from A, through the ſaid mark, as A C, ſhall with A B, form the angle required, *viz.* 48 degrees.

If the line had been to make an angle with A B, at the point B; then the centre muſt have been laid on B, and the diviſions counted from the left had towards the right.

EXAM. II. *To find the number of degrees which meaſure the angle* A B C. Fig. 9.

APPLY the blank edge of the protractor to the line A B, ſo that the centre ſhall fall on the point B; then will the line B C cut the limb of the Inſtrument in the number expreſſing the degrees which meaſure the given angle; which in this *example* is 125 degrees, counting from the left hand towards the right.

PROBLEM. V.

From any given point A, *in a line* A B. *to erect a perpendicular.* Fig. 10.

LAY the protractor acroſs the line A B in ſuch a manner that the centre on the blank edge, and the diviſion numbered with 90, on the limb, may both be cut by the given line; then keeping the Ruler in this poſition, ſlide it along the line, till one of theſe points touch the given point A, draw the line C A, and it will be perpendicular to A B.

IN

In the same manner, a line may be let fall from a given point, perpendicular to a given line.

PROBLEM VI.

In a circle given to describe any regular Polygon. Sup. an octogon. Fig. 11.

Con Apply the blank edge of the *protractor* to the diameter of the *Circle*, so that their centres shall coincide; set off a number of degrees equal to an angle at the centre of that *polygon*, (viz. 45.) and through that mark draw a radius; then shall the chord of the arc expressing those degrees, be the side of the intended *polygon*; which chord taken between the compasses, will divide the circumference into as many equal parts as the *polygon* has sides, *viz.* 8.

A TABLE,

A TABLE, *shewing the Angles at the Centers and Circumferences of regular* Polygons *from three to twelve Sides inclusive.*

Names.	Sides.	Angles at Center		Angles at Cir.	
Trigon	3	120°	00′	60°	00′
Square	4	90	00	90	00
Pentagon	5	72	00	108	00
Hexagon	6	60	00	120	00
Heptagon	7	51	$25\frac{5}{7}$	128	$34\frac{2}{7}$
Octagon	8	45	00	135	00
Nonagon	9	40	00	140	00
Decagon	10	36	00	144	00
Endecagon	11	32	$43\frac{7}{11}$	147	$16\frac{4}{11}$
Dodecagon	12	30	00	150	00

THIS table is constructed, by dividing 360, the degrees in a circumference, by the number of sides in each polygon; and the quotients are the angles at the centers; the angle at the center substracted from 180 degrees, leaves the angle at the circumference.

PROBLEM VII.

Upon a given right line AB, *to describe any regular polygon,* Fig. 12.

CON-

CONSTRUCTION. From the ends of the given line, draw the lines AD, BC; so that the angles BAD, ABC, may each be equal to the angle at the circumference in that polygon; make AD, BC, each equal to AB; from the points D and C, draw lines that shall make with DA, CB, angles equal to the former; make these lines each equal to AB; and so continue, till a polygon is form'd of as many sides as required.

EXAM. I *Upon the line* AB *to describe an hexagon.* Fig. 12.

DRAW AD, BC, so that the angles DAB, ABC, may be each 120 degrees; make DA, BC, each equal to AB: also, make the angles FDA, ECB, each equal to 120 degrees, and make DF, CE, each equal to AB; draw FE and 'tis done.

OR it may be done by help of the parallel ruler, there being an even number of sides. Thus,

HAVING form'd the three sides DA, AB, BC, as before directed, through D, draw DF parallel to BC; make DF equal to AB; through F draw FE parallel to AB: make FE equal to AB and join CE.

EXAM. II. *Upon the line* AB *to describe a pentagon.* Fig. 13.

DRAW CA, DB, that each may make with AB, an angle of 108 degrees. Make CA, DB, each equal to AB; on the points C and D, with the compasses opened to the distance AB, discribe arcs to cross each other in E; draw EC and CD, and 'tis done.

In any regular polygon, having found all the ſides but two, as above directed; thoſe may be found as the laſt two in the pentagon were.

Sect. VIII.

Of the Plain Scale.

THE lines generally drawn on the plain ſcale, are theſe following:

I. Lines of equal parts.
II. ——— Chords.
III. ——— Rhumbs.
IV. ——— Sines.
V. ——— Tangents.
VI. ——— Secants.
VII. ——— Half Tangents.
VIII. ——— Longitude.
IX. ——— Latitude.
X. ——— Hours.
XI. ——— Inclinations.

Of the Lines of equal Parts.

LINES of equal parts are of two ſorts, *viz.* ſimply divided, and diagonally divided.

I. *Simply divided.* Draw 3 lines parallel to each other, at unequal diſtances, (Fig. 14.) and of any convenient length; divide this length into what number of equal parts is thought neceſſary, allowing ſome certain number of theſe parts to an inch, ſuch as 2, 2 $\frac{1}{2}$,

3,

3, 3½, 4, 4½, *&c.* which divisions distinguish by lines drawn across the three parallels. Divide the left hand division into 10 equal parts, which distinguish by lines drawn across the lower parallels only; but, for distinction sake, let the 5th division be somewhat longer than the others: And it may not be inconvenient to divide the same left-hand division into 12 equal parts, which are laid down on the upper parallel line, having the 3d, 6th, and 9th divisions distinguished by longer strokes than the rest, whereof that at the 6th division make the longest.

There are, for the most part, several of these simply divided scales put on rulers one above the other, with numbers on the left hand, shewing in each scale, how many equal parts an inch is divided into; such as 20, 25, 30, 35, 40, 45, *&c.* and are severally used, as the plan to be expressed should be larger or smaller.

The use of these lines of equal parts, is to lay down any line expressed by a number of two places or denominations, whether decimally, or duodecimally divided; as leagues, miles, chains, poles, yards, feet, inches, *&c.* and their tenth parts, or twelfth parts: Thus, if each of the divisions be reckoned 1, as 1 league, mile, chain, *&c.* then each of the subdivisions will express $\frac{1}{10}$ part thereof; and if each of the large divisions be called 10, then each small one will be 1; and if the large divisions be 100, then each small one will be 10, *&c.*

THEREFORE to lay off a line 8 $\frac{7}{10}$, 87, or 870 parts, let them be leagues, miles, chains, &c. set one point of the compasses on the 7th of the small divisions, counting from the right hand towards the left, and open the compasses, till the other point falls on the 8th of the large divisions, counting from the left hand towards the right, then are the compasses opened to express a line of 8 $\frac{7}{10}$, 87 or 870 leagues, miles, chains, &c. and bears such proportion in the plan, as the line measured does to the thing represented.

BUT if a length of feet and inches was to be expressed, the same large divisions may represent the feet, but the inches must be taken from the upper part of the first division, which (as before noted) is divided into 12 equal parts.

THUS, if a line of 7 feet 5 inches was to be laid down; set one point of the compasses on the 5th division among the 12, counting from the right hand towards the left, and extend the other to 7, among the large divisions, and that distance laid down in the plan, shall express a line of 7 feet 5 inches: And the like is to be understood of any other dimensions.

II. *Diagonally divided.* Draw eleven lines parallel to each other, and at equal distances; divide the upper of these lines into such a number of equal parts, as the scale to be expressed is intended to contain, and from each of these divisions draw perpendiculars through the eleven parallels, (Fig. 15.) subdivide the first of these divisions into 10 equal parts, both

both in the upper and lower lines; each of these subdivisions may be also subdivided into 10 equal parts by drawing diagonal lines; *viz.* from the 10th below, to the 9th above; from the 9th below, to the 8th above; from the 8th below, to the 7th above, *&c.* till from the 1st below to the 0th above, so that by these means one of the primary divisions on the scale, will be divided into 100 equal parts.

THERE are generally two diagonal scales laid on the same plane or face of the ruler, one being commonly half the other. (Fig. 15.)

THE use of the diagonal scale is much the same with the simple scale; all the difference is, that a plan may be laid down more accurately by it: Because in this, a line may be taken of three denominations; whereas from the former, only two could be taken.

NOW from this construction it is plain, if each of the primary divisions represent 1, each of the first subdivisions will express $\frac{1}{10}$ of 1; and each of the second subdivisions, (which are taken on the diagonal lines, counting from the top downwards) will express $\frac{1}{10}$ of the former subdivisions, or a 100th of the primary divisions; and if each of the primary divisions express 10, then each of the first subdivisions will express 1, and each of the 2d, $\frac{1}{10}$; and if each of the primary divisions represent 100, then each of the first subdivisions will be 10; and each of the 2d will be 1, *&c.*

THEREFORE to lay down a line, whose length is express'd by 347, 34 $\frac{7}{10}$ or 3 $\frac{47}{100}$ whether leagues, miles, chains, *&c.*

On the diagonal line, join'd to the 4th of the first subdivisions, count 7 downwards, reckoning the distance of each parallel 1; there set one point of the compasses, and extend the other, till it falls on the intersection of the third primary division with the same parallel in which the other foot rests, and the compasses will then be opened to express a line of 347, $34\frac{7}{10}$; or $3\frac{47}{100}$. *&c.*

Those who have frequent occasion to use scales, perhaps will find, that a rular with the 20 following scales on it, *viz.* 10 on each face, will suit more purposes than any set of simply divided scales hitherto made public, on one ruler.

One Side	The divisions	10, 11, 12, $13\frac{2}{3}$, 15, $16\frac{2}{3}$, 18, 20, 22, 25,
Other Side	to an inch	28, 32, 36, 40, 45, 50, 60, 70, 85, 100,

The left hand primary division, to be divided into 10 and 12 and 8 parts; for these subdivisions are of great use in drawing the parts of a fortress, and of a piece of cannon.

It will here be convenient to shew, how any plan expressed by right lines and angles, may be delineated by the scales of equal parts, and the protractor.

PROBLEM. VIII.

Three adjacent things in any right line, triangle being given, to form the plan thereof.

Examp. *Suppose a triangular field*, ABC, (Fig. 16.) the side AB=327 yards; AC=208 yards; and the angle at A=$44\frac{1}{2}$ degrees.

Cons.

Cons. Draw AB at pleasure; from the scale take 327, and lay it from A to B; set the center of the protractor to the point A, lay off $44\frac{1}{2}$ degrees, and by that mark draw AC: Take from the same scale 208, lay it from A to C, and join CB; so shall the parts of the triangle ABC, in the plan, bear the same proportion to each other, as the real parts in the field does.

If two angles and the side contained between them were given, draw a line to express the side; (as before.) at the ends of that line, point off the angles, as observed in the field; lines drawn from the ends of the given line through those marks, shall form a triangle similar to that of the field.

PROBLEM. IX.

Five adjacent things, sides and angles, in a right lin'd quadrilateral, being given, to lay down the plan thereof, Fig. 17.

Examp. *Given* $\angle A = 70^\circ$; $AB = 215$ links; $\angle B = 115^\circ$; $BC = 596$ links; $\angle C = 114^\circ$.

Draw AD at pleasure; from A draw AB, so as to make with AD an angle of 70°: Make $AB = 215$; (taken from the scales.) from B, draw BC, to make an angle of 115°: Make $BC = 596$; from C, draw CD, to make an angle of 114°, and by the intersection of CD with AD, a quadrilateral will be form'd similar to the figure in which such measures could be taken as are expressed in the example.

If 3 of the things were sides, the plan might be formed with equal ease.

Following the same method, a figure of many more sides may be delineated; and in this manner, or some other like to it, do surveyors make their plans of surveys.

The Construction of the remaining Lines of the Plain Scale.

Preparation. Fig. 18.

Describe a circumference with any convenient radius, and draw the diameters AB, DE, at right angles to each other; continue BA at pleasure towards E; through D, draw DG parallel to BF; and draw the chords BD, BE, AD, AE. Circumscribe the circle with the square HMN, whose sides HM, MN, shall be parallel to AB ED.

I. *To construct the Line of Chords.*

Divide the arc AD into 90 equal parts; mark the 10th divisions with the figures 10, 20, 30, 40, 50, 60, 70, 80, 90; on D, as a center, with the compasses, transfer the several divisions of the quadrantal arc, to the chord AD, which marked with the figures corresponding, will become a line of chords.

Note, In the construction of this, and the following scales, only the primary divisions are drawn, the intermediate ones are omitted, that the figure may not appear too much crouded.

II.

II. *The Line of Rhumbs.*

Divide the arc BE into 8 equal parts, which mark with the figure 1, 2, 3, 4, 5, 6, 7, 8 ; and each of those into quarters ; on B, as a center, transfer the divisions of the arc to the chord BE, which marked with the corresponding figures, will be a line of rhumbs.

III. *The Line of Sines.*

Through each of the divisions of the arc AD, draw right lines parallel to the radius AC ; and CD will be divided into a line of sines which are to be numbered from C to D for the right sines ; and from D to C for the versed sines. The versed sines may be continued to 180 degrees by laying the divsions of the radius CD, from C to E.

IV. *The Line of Tangents.*

A ruler on C, and the several divisions of the arc AD, will intersect the line DG, which will become a line of tangents, and are to be figured from D to G with 10, 20, 30, 40, *&c.*

V. *The Line of Secants.*

From the Center C, the line of tangents being transfered to the line AF, will give the divisions of the line of secants ; which must be numbered from A towards E, with 10, 20, 30, *&c.*

VI. *The Line of Half-Tangents (or the Tangents of half the Arcs).*

A ruler on E, and the ſeveral diviſions of the arc AD, will interſect the radius CA, in the diviſions of the half tangents; mark theſe with the correſponding figures of the arc AD.

VII. *The Lines of Longitude.*

Divide AH, into 60 equal parts; through each of theſe diviſions, parallels to the radius AC, will interſect the arc AE, in as many points; from the center A, the diviſions of the arc AE, being transferred to the chord AE, will give the diviſions of the line of longitude.

VIII. *The Line of Latitude.*

A ruler on A, and the ſeveral diviſions of the ſines CD, will interſect the arc BD, in as many points; on B as a center, transfer the interſection of the arc BD, to the line BD; number the diviſions from B to D, with 10, 20, 30, &c. to 90; and BD will be a line of latitude.

IX. *The Line of Hours.*

Bisect the quadrantal arcs BD, BE, in a, b; divide the arc a b into 6 equal parts, (which gives 15 degrees for each hour.) and each of theſe into 4 others; (which will give the quarters.) A ruler on C, and the ſeveral diviſions

of

of the arc a b, will interſect the line MN in the Hour, *&c.* points, which are to be mark'd as in the figure.

X. *The Line of Inclinations of Meridians.*

Bisect the arc EA in c; divide the quadrantal arc b c into 90 equal parts; lay a ruler on C and the ſeveral diviſions of the arc bc, and the interſections of the line H M will be the diviſions of a line of inclinations of meridians.

Sect. IX.

The uſes of ſome of the Lines on the Plain Scale.

I. *Of the Line of Chords.*

The chief uſe of the line of chords is to lay down a propoſed angle, or to meaſure an angle already laid down. Thus, to draw a line AC, that ſhall make with the line AB an angle containing a given number of degrees. (ſuppoſe 36) Figure 19.

On A, as center, with the chord of 60 degrees, deſcribe the arc BC; on this arc, lay the chord of the given number of degrees from the interſection B, to C; draw AC, and the angle BAC will contain the given number of degrees.

Note, Degrees taken from the chords are always to be counted from the beginning of the Scale.

The

The degrees contain'd in an angle already laid down, may be meaſured thus. Fig. 19.

On A as a center, deſcribe an arc BC with the chord of 60 degrees; the diſtance BC, meaſured on the chords, will give the number of degrees contain'd in the angle **BAC**.

If the number of degrees are more than 90; they muſt be taken from, or meaſured by the chords, at twice; thus if 140 degrees were to be protracted, 70° may be taken from the chords, and thoſe degrees laid of twice upon the arc deſcrib'd with a chord of 60 degrees.

II. *Of the Line of Rhumbs.*

Their uſe is to delineate or meaſure a ſhip's courſe; which is the angle made by a ſhip's way and the meridian.

Now having the points and $\frac{1}{4}$ points of the compaſs contain'd in any courſe; draw a line AB (fig. 19.) for the meridian; on A as center, with the chord of 60° deſcribe an arc BC; take the number of points and $\frac{1}{4}$ points from the ſcale of rhumbs, counting from 0, and lay this diſtance on the arc BC, from the interſection B to C; draw AC, and that ſhall repreſent the ſhip's courſe.

III. *The uſe of the Line of Longitude.*

If any two meridians be diſtant one degree or 60 geographical miles, under the equator, their diſtance will be leſs than 60 miles in any latitude between the equator and the pole.

Now

Now let the line of longitude be put on the ſcale cloſe to the line of chords, but inverted; that is, let 60° in the ſcale of longitude be againſt 0° in the chords, and 0° degrees longitude againſt 90° chords. Then mark any degree of latitude counted on the chords; and oppoſite thereto, on the line of longitude, will be the miles contain'd in one degree of longitude, in that latitude.

THUS 57,95 miles, make 1 degree longitude in the latitude of 15 degrees; 45,97 miles, in latitude 40 degrees; 36,94 miles, in latitude 52 degrees; 30 miles, in latitude 60 degrees, *&c.*

BUT as the fractional parts are not very obvious on ſcales, here follows a table ſhewing the miles in one degree of longitude to every degree of latitude.

This table is computed upon the ſuppoſition of the Earth being ſpherical.

A TABLE,

A TABLE, *shewing the Miles in one Degree of Longitude to every Degree of Latitude.*

D.L.	Miles	D.L.	Miles.	D.L.	Miles.
1	59,99	31	51,43	61	29,09
2	59,96	32	50,88	62	28,17
3	59,62	33	50,32	63	27,24
4	59,85	34	49,74	64	26,30
5	59,77	35	49,15	65	25,36
6	59,67	36	48,54	66	24,41
7	59,56	37	47,92	67	23,44
8	59,42	38	47,28	68	22,48
9	59,26	39	46,63	69	21,50
10	59,09	40	45,97	70	20,52
11	58,89	41	45,28	71	19,53
12	58,69	42	44,59	72	18,54
13	58,46	43	43,88	73	17,54
14	58,22	44	43,16	74	16,54
15	57,95	45	42,43	75	15,53
16	57,67	46	41,68	76	14,52
17	57,38	47	40,92	77	13,50
18	57,06	48	40,15	78	12,48
19	56,73	49	39,36	79	11,45
20	56,38	50	38,57	80	10,42
21	56,02	51	37,76	81	9,38
22	55,63	52	36,94	82	8,35
23	55,23	53	36,11	83	7,32
24	54,81	54	35,27	84	6,28
25	54,38	55	34,41	85	5,23
26	53,93	56	33,55	86	4,18
27	53,46	57	32,68	87	3,14
28	52,96	58	31,79	88	2,09
29	52,47	59	30,90	89	1,07
30	51,96	60	30,00	90	0,00

THE

The uses of the scales of sines, tangents, secants, and half tangents, are to find the poles and centers of the several circles represented in the orthographical and stereographical projection of the sphere; which are reserved until the explanation and use of the lines of the same name on the sector are shewn.

The lines of latitudes, hours, and inclinations of meridians, are applicable to the practice of dialling; on which there are several treatises extant, which may be consulted.

Sect. X.

Of the Sector.

A Sector is a figure form'd by two radius's of a circle, and that part of the circumference comprehended between the two radius's.

The instrument called a sector, consists of two rulers moveable round an axis or joint, from whence several scales are drawn on the faces of the rulers.

The two rulers are called legs, and represent the radii, and the middle of the joint expresses the center.

The scales generally put on sectors, may be distinguished into single, and double.

The single scales are such as are commonly put on plain scales, and from whence dimensions or distances are taken as have been already directed.

The

THE double ſcales are thoſe which proceed from the center; each ſcale is laid twice on the ſame face of the inſtrument, *viz.* once on each leg: From theſe ſcales, dimenſions or diſtances are to be taken, when the legs of the inſtrument are in an angular poſition, as will be ſhewn hereafter.

The Scales commonly put on the beſt Sectors, are

Single	1	a line of	Inches, each Inch divided into 8 and 10 parts.	
	2		Decimals, containing an 100 parts.	
	3		Chords,	mark'd Cho.
	4		Sines,	Sin.
	5		Tangents,	Tang.
	6		Rhumbs,	Rum.
	7		Latitude,	Lat.
	8		Hours,	Hou.
	9		Longitude,	Lon.
	10		Inclin. Merid.	In. Me.
	11		the Logs. of Numbers,	Num.
	12		the Logs. of Sines,	Sin.
	13		the Logs. of Verſed Sines,	V. Sin.
	14		the Logs. of Tangents,	Tan.

Double	1	a line of	Lines,	mark'd Lin.
	2		Chords,	Cho.
	3		Sines,	Sin.
	4		Tangents to 45°	Tan.
	5		Secants,	Sec.
	6		Tangents to above 45°	tan.
	7		Polygons,	Pol.

THE manner in which theſe ſcales are diſpoſed of on the ſector, is beſt ſeen in the plate fronting the title page.

THE ſcales of lines, chords, ſines, tangents, rhumbs, latitudes, hours, longitude, incl.

incl. merid. may be uſed, whether the inſtrument is ſhut or open, each of theſe ſcales being contained on one of the legs only. The ſcales of inches, decimals, log. numbers, log. ſines, log. verſed ſines, and log. tangents, are to be uſed with the ſector quite opened, part of each ſcale lying on both legs.

The double ſcales of lines, chords, ſines, and lower tangents, or tangents under 45 degrees, are all of the ſame radius or length; they begin at the center of the inſtrument, and are terminated near the other extremity of each leg; *viz.* the lines at 10, the chords at 60, the ſines at 90, and the tangents at 45; the remainder of the tangents, or thoſe above 45°, are on other ſcales beginning at $\frac{1}{4}$ of the length of the former, counted from the center, and marked with 45, and run to about 76 degrees.

The ſecants alſo begin at the ſame diſtance from the center, where they are marked with 10, and are from thence continued to as many degrees as the length of the ſector will allow, which is about 75°.

The angles made by the double ſcales of lines, of chords, of ſines, and of tangents to 45 degrees, are always equal.

And the angles made by the ſcales of upper tangents, and of ſecants, are alſo equal; and ſometimes theſe angles are made equal to thoſe made by the other double ſcales.

The ſcales of polygons are put near the inner edge of the legs, their beginning is not ſo far removed from the center, as the

60 on the chords is: Where these scales begin, they are mark'd with 4, and from thence are figured backwards, or towards the center, to 12.

From this disposition of the double scales, it is plain, that those angles which were equal to each other, while the legs of the sector were close, will still continue to be equal, although the sector be opened to any distance it will admit of.

Sect. XI.

Of the Construction of the Single Scales.

I. *The Scale of Inches.*

THIS scale, which is laid close to the edge of the sector, and sometimes on the edge, contains as many inches as the instrument will receive when opened: Each inch is divided into 8 equal parts, and also into 10 equal parts.

II. *The Decimal Scale.*

This scale lies next to the scale of inches; it is of the same length of the sector, (as suppose a foot) and is divided into 10 equal parts, or primary divisions; and each of these into 10 other equal parts; so that the whole (foot) is divided into 100 equal parts.

III.

III. *The Scales of Chords, Rhumbs, Sines, Tangents, Hours, Latitudes, Longitudes, and Inclination of Meridians;*

ARE ſuch as have been already deſcribed in the account of the plane ſcale.

IV. *The Scale of Logarithmic Numbers.*

THIS ſcale, commonly called the artificial numbers, and by ſome the *Gunter*'s ſcale, or *Gunter*'s * line, is a ſcale expreſſing the logarithms of common numbers, taken in their natural order. To lay down the diviſions in the beſt manner, there is neceſſary a good table of logarithms, (ſuppoſe *Sherwin*'s,) and a ſcale of equal parts, accurately divided, and of ſuch a length, that 20 of the primary diviſions ſhall make the whole length of the intended ſcale of numbers, or logarithm ſcale.

The Construction.

1. FROM the ſcale of equal parts, take the firſt 10 of the primary diviſions, and lay this diſtance down twice on the log. ſcale, making two equal intervals; marking the firſt point 1, the ſecond 1, (or rather 10) and the third 10, (or rather 100.)

* From Mr. *Edmund Gunter*, the Inventor: Aſtronomy-Profeſſor in *Greſham-College*, Anno 1624.

2. From the ſcale of equal parts, take the diſtances expreſſed by the logs. of the numbers 2, 3, 4, 5, 6, 7, 8, 9, reſpectively, (rejecting the indices :) lay theſe diſtances on each interval of the log. ſcale, between the marks 1 & 10, 10 & 100, reckoning each diſtance from the beginning of its interval, *viz.* from 1, and from 10, and mark theſe diſtances with the figures 2, 3, 4, 5, 6, 7, 8, 9, in order.

3. The diſtances expreſſing the logs. of the numbers between 10 & 20, 20 & 30, 30 & 40, 40 & 50, 50 & 60, 60 & 70, 70 & 80, 80 & 90, 90 & 100, (rejecting the indices) are to be taken from the ſcale of equal parts, and laid on the log. ſcale, in each of the primary intervals, between the marks 1 & 2, 2 & 3, 3 & 4, 4 & 5, 5 & 6, 6 & 7, 7 & 8, 8 & 9, 9 & 10, reſpectively; reckoning each diſtance from the beginning of its reſpective primary interval.

4. The laſt ſubdiviſions of the ſecond primary interval are to be divided into others, as many as the ſcale will admit of, which is done by laying down the logarithms of ſuch intermediate diviſions, as it ſhall be thought proper to introduce.

V. *The Scale of Logarithm Sines.*

1. From the ſcale of equal parts, take the diſtances expreſſed by the arithmetical complements of the logarithmic ſines, (or the ſecants of the complements) of 80, 70, 60, 50, 40, 30, 20, 10, degrees reſpectively;

re-

rejecting the indices; and these distances, lay on the scale of log. sines, reckoning each from the mark intended to express 90 degrees.

2. In the same manner, lay of the degrees under 10: and also, the degrees intermediate to those of 10, 20, 30, *&c.*

3. Lay down as many of the multiples of 5 minutes, as may conveniently fall within the limits of such degrees as will admit of such subdivisions of minutes.

VI. *The Scale of Logarithmic Tangents.*

1. This scale, to 45 degrees, is constructed in every particular, like that of the log. sines: using the arithmetical complements of the log. tangents.

2. The degrees above 45, are to be counted backwards on the scale: Thus 40 on the scale, represents both 40 degrees, and 50 degrees; 30 on the scale, represents both 30 degrees, and 60 degrees; and the like of the other mark'd degrees, and also of their intermediate ones.

VII. *The Logarithmic versed Sines.*

1. From the scale of equal parts, take the arithmetical complements of the logarithm cosines, (or the secants of the complements) of 5, 10, 15, 20, 25, 30, 35, 40, *&c.* degrees; (rejecting the indices,) and the double of these distances, respectively, laid on the scale (intended) for the log. versed sines, will

give the divisions expressing 10, 20, 30, 40, 50, 60, 70, 80, *&c.* degrees; to as many as the length of the scale will take in.

2. BETWEEN every distance of 10 degrees, introduce as many degrees, $\frac{1}{2}$ degrees; $\frac{1}{3}$ degrees; $\frac{1}{4}$ degrees, *&c.* as the intervals will admit.

THE scales of the logarithms of numbers, sines, versed sines, and tangents, should have one common termination to one end of each scale; that is the 10 on the numbers, the 90 on the sines, the 0 on the versed sines, and the 45 on the tangents, should be opposite to each other: The other end of each of the scales of sines, versed sines, and tangents, will run out beyond the beginning (mark'd 1) of the numbers; nearly opposite to which, will be the divisions representing 35 minutes on the sines and tangents, and 168 $\frac{1}{2}$ degrees, on the versed sines.

SECT. XII.

Of the Construction of the double scales.

I. *Of the Line of Lines.*

THIS is only a scale of equal parts, whose length is adapted to that of the legs of the sector: Thus in the six inch sector, the length is about 5 $\frac{3}{4}$ inches.

THE length of this scale is divided into 10 primary divisions; each of these into 10 equal secondary parts; and each secondary division, into 4 equal parts.

II. *Of*

II. *Of the Line of Sines.*

1. Make the whole length of this ſcale, equal to that of the line of lines.

2. From the ſcale of the line of lines, take off ſeverally, the parts expreſs'd by the numbers in the tables (ſuppoſe *Sherwin's*) of the natural ſines, correſponding to the degrees, or to the degrees and minutes, intended to be laid on the ſcale.

3. Lay down theſe diſtances ſeverally on the ſcale, beginning from the center; and this will expreſs a ſcale of natural ſines.

Exam. *To lay down* 35° 15′; *whoſe natural Sine found in the Tables is* 57714, *&c.*

Take this number as accurately as may be, from the line of lines, counting from the center; and this diſtance will reach from the beginning of the ſines, at the center of the inſtrument, to the diviſion expreſſing 35° 15′; and ſo of the reſt.

In ſcales of this length, 'tis cuſtomary to lay down diviſions, expreſſing every 15 minutes, from 0 degrees to 60 degrees; between 60 and 80 degrees, every half degree is expreſs'd; then every degree to 85; and the next, is 90 degrees.

Of the Scale of Tangents.

The length of this ſcale is equal to that of the line of lines, and the ſeveral diviſions thereon (to 45 degrees) are laid down from the tables and line of lines, in the ſame man-

ner as has been described in the sines; observing to use the natural tangents in the tables.

IV. *Of the Scale of upper Tangents.*

This scale is to be laid down, by taking $\frac{1}{4}$ of such of the natural tabular tangents above 45 degrees, as are intended to be put on the scale.

ALTHOUGH the position of this scale on the sector respects the center of the instrument, yet its beginning, at 45 degrees, is distant from the center, $\frac{1}{4}$ of the length or radius of the lower tangent.

V. *Of the Scale of Secants.*

THE distance of the beginning of this scale, from the center, and the manner of laying it down, is just the same as that of the upper tangents; only in this, the tabular secants are to be used.

VI. *Of the Scale of Chords.*

1. MAKE the length of this scale, equal to that of the sines; and let the divisions to be laid down, express every 15 minutes from 0 degrees to 60 degrees.

2. TAKE the length of the sine of half the degrees and minutes, for every division to be laid down, (as before directed in the scale of sines;) and twice this length, counted from the center, will give the divisions required.

THUS

THUS, twice the length of the ſine 18° 15′, will give the chord of 36° 30′; and in the ſame manner for the reſt.

VII. *Of the Scale of Polygons.*

THIS ſcale only takes in the ſides of the polygons from 4 to 12 ſides incluſive: The diviſions are laid down, by taking the lengths of the chords of the angles at the center of each polygon; and this diſtance is laid from the center of the inſtrument.

BUT the diviſions are to be taken from a ſcale of chords where the length of 90 degrees, is equal to that of 60 degrees of the double ſcale of chords on the ſector.

IN the place of ſome of the double ſcales here deſcribed, there are found other ſcales on the old ſectors, and alſo on ſome of the modern *French* ones, ſuch as, ſcales of ſuperficies, of ſolids, of inſcrib'd bodies, of metals, *&c.* But theſe ſeem to be juſtly left out on the ſectors, as now conſtructed, to make room for others of more general uſe: However, theſe ſcales, and ſome others, of uſe in gunnery, ſhall hereafter be deſcribed in a tract on the uſe of the gunners callipers.

SECT. XIII.

Of the Uſe of the Double Scales.

IN the following account of the uſes, as there will frequently occur the terms *lateral diſtance*, and *tranſverſe diſtance*; it will be pro-

per

per to explain what is meant by thoſe terms.

Lateral diſtance, is a diſtance taken by the compaſſes on one of the ſcales.

Tranſverſe diſtance, is the diſtance taken between any two correſponding diviſions of the ſcales of the ſame name, the legs of the ſector being in an angular poſition.

Some uſes of the Line of Lines.

PROBLEM X.

To two given lines AB = 2, BC = 6; *to find a third proportional.* Fig 20.

1. TAKE between the compaſſes, the lateral diſtance of the ſecond term, (*viz.* 6.)

2. SET one point on the diviſion expreſſing the firſt term (*viz.* 2.) on one leg, and open the legs of the ſector till the other point will fall on the correſponding diviſion on the other leg.

3. KEEP the legs of the ſector in this poſition; take the tranſverſe diſtance of the ſecond term, (*viz.* 6.) and this diſtance is the third term required.

4. THIS diſtance meaſured laterally, beginning from the center, will give the number expreſſing the meaſure of the third term.

Note, If the legs of the ſector will not open ſo far as to let the lateral diſtance of the ſecond term fall between the diviſions expreſſing the firſt term; then take $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, or any aliquot part of the ſecond term, (ſuch as will conveniently fall within the opening of the

the ſector) and make ſuch part, the tranſverſe diſtance of the firſt term; then if the tranſverſe diſtance of the ſecond term be multiplied by the denominator of the part taken of the ſecond term, the product will give the third term.

PROBLEM XI.

To three given lines AB = 3, BC = 7, CD = 10; *to find a fourth proportional*, Fig. 20.

OPEN the legs of the ſector, untill the tranſverſe diſtance of the firſt term, (3) be equal to the lateral diſtance of the ſecond term, (7) or to ſome part thereof; then will the tranſverſe diſtance of the third term, (10) give the fourth term, (23 $\frac{1}{3}$) required; or, ſuch a ſubmultiple thereof as was taken of the ſecond term.

FROM his problem is readily deduced, how to increaſe or diminiſh a given line, in any aſſign'd proportion.

EXAM. *To diminiſh a line of* 4 *inches, in the proportion of* 8 *to* 7.

1. Open the ſector untill the tranſverſe diſtance of 8 & 8, be equal to the lateral diſtance of 7.

2. MARK the point to where 4 inches will reach, as a lateral diſtance taken from the center.

3. THE tranſverſe diſtance, taken at that point, will be the line requir'd.

IF the given line, ſuppoſe 12 inches, ſhould be too long for the legs of the ſector, take $\frac{1}{2}$, or $\frac{1}{3}$, or $\frac{1}{4}$, *&c.* part of the given line for the lateral diſtance; and the correſponding tranſ-

transverse distance, taken twice, or thrice, or four times, *&c.* will be the line required.

PROBLEM XII.

To divide a given line into any propos'd number of equal parts: (suppose 9.)

MAKE the length of the given line, or some known part thereof, a transverse distance to 9 & 9: Then will the transverse distance of 1 & 1, be the $\frac{1}{9}$ part thereof; or such a submultiple of the $\frac{1}{9}$ part, as was taken of the given line.

OR the $\frac{1}{9}$ part, will be the difference between the given line, and the transverse distance of 8 & 8.

THE latter of these methods is to be preferred when the part required falls near the center of the instrument.

To this problem may be referred the method of making a scale of a given length, to contain a given number of equal parts.

THE practice of this is very useful to those who have occasion to take copies of surveys of lands; draughts of buildings, whether civil or military; and in every other case, where drawings are to be made to bear a given proportion to the things they represent.

EXAM. *Suppose the scale to the map of a survey is* 6 *inches long, and contains* 140 *poles; required to open the sector so, that a corresponding scale may be taken from the line of lines.*

SOLUTION. Make the transverse distance 7 & 7 (or 70 & 70, *viz.* $\frac{140}{2}$) equal to 3 inches

inches; $(=\frac{6}{2})$ and this position of the line of lines will produce the given scale.

If it was required to make a scale of 140 *poles, and to be only* 2 *inches long.*

SOLUTION. Make the transverse distance of 7 & 7 equal to 1 inch, and the scale is made.

EXAMP. II. *To make a scale of* 7 *inches long, contain* 180 *fathoms.*

SOL. Make the transverse distance of 9 & 9 equal to $3\frac{1}{2}$ inches, and the scale is made.

EXAM. III. *To make a scale which shall express* 286 *yards, and be* 18 *inches long.*

SOL. Make the $\frac{1}{3}$ of 18 inches (or 6 inch) a transverse distance to the $\frac{1}{3}$ of 286 $(=95\frac{1}{3})$ and the scale is made.

OR, Make the $\frac{1}{4}$ of 18 inches $(=4\frac{1}{2}$ inches) a transverse distance to $\frac{1}{4}$ of 286 $(=71\frac{1}{2})$ and the scale is made.

PROBLEM XIII.

The use of the line of lines, in drawing the orders of civil architecture.

IT is customary among architects to estimate the heights and projections of all the parts of every order, by the diameter of the column at bottom, which they call a module, and is supposed to consist of 60 equal parts, which are called minutes.

IN the three following tables are contained the heights and projections of the parts of each order, according to the proportions given by *Palladio*; the orders of this archi-

tect

tect were chosen, because the *English*, at present, are more fond of copying his productions. than those of any other architect.

The first table serves for the pedestal, the second for the column, and the third for the entablature, of each order. Each table is divided into 7 principal columns: In the first, beginning at the left hand, is contained the names of the primary divisions; in the second those of the several divisions and members in the orders; and the other five titled with *Tuscan*, *Doric*, *Ionic*, *Corinthian*, *Roman*, contain the numbers expressing the altitudes, and projections taken from the axis, or middle of the column, of the several members belonging to their corresponding orders.

The column containing each order, is divided, first into two other columns, one shewing the altitudes, and signed Alt. and the other, the projections, and sign'd Proj. Each of these is also divided into two other columns, one containing modules, and mark'd Mod. and the other, the minutes and parts, and mark'd Min.

Under the table of the pedestal there is another table, shewing the general proportions for the heights of the orders.

In each of the orders of architecture, the height of the order, and the diameter of the column, have a constant relation to one another.

Therefore, if the diameter of the column be given, the height of the order is given also: And having determin'd by what

scale

ſcale the order is to be drawn, ſuch as $\frac{1}{2}$ inch, 1 inch, 2 inches, *&c.* to a foot or yard, *&c.* Take from ſuch ſcale, the part or parts expreſſing the diameter of the column, and make this extent a tranſverſe diſtance to 6 & 6, (*i. e.* 60 & 60) on the ſcales of lines, and the ſector will be opened ſo, that the ſeveral proportions of the order may be taken from it.

EXAMP. *Suppoſe the diameter of a column is to be* 18 *inches; and the drawing of the order is to be delineated from a ſcale of an inch to a foot: that is, the diameter of the column in the drawing is to be an inch and half.*

MAKE the tranſverſe diſtance of 6 & 6, on the ſcales of lines, equal to 1 $\frac{1}{2}$ inch, and the ſector is fitted for the ſcale.

IF the height of the order is given, divide this height, by the height of the order in the table; and the quotient will be the diameter of the column.

EXAM. *What muſt be the diameter of the column in the* Ionic *order, when the whole height of the order is fixed at* 18 *feet* 6 *inches.*

THE height of the order in the table is 13 mo. 29 mi. $\frac{1}{4} = 13\frac{29,25}{60} = 13,4875$ modules: And 18 f. 6 in. $= 18,5$ feet. Th. $\frac{18,5}{13,4875} = 1,3709$ feet $= 1$ f. $4\frac{1}{2}$ inches nearly: And the ſector may be fitted to this, as before directed, according to the intended ſize of the draught.

To

To delineate an Order by these Tables.

HAVING determined the diameter of the column at bottom, and set the sector to the intended scale, draw a line to represent the axis or middle of the order.

ON this line, lay the parts for the heights of the pedestal, column, and entablature, taken from the table of general proportions.

WITHIN each of these parts respectively, lay the several altitudes taken from the tables of particulars, under the word Alt. Through each of the points mark'd on the axis, draw lines perpendicular to the axis, or draw one line perpendicular, and the others parallel thereto.

ON the lines drawn perpendicular to the axis, lay the projections corresponding to the respective altitudes; these projections are to be laid on both sides of the axis, for the pedestal and column; and only on one side, for the entablature, join the extremities of the projections with such lines as are proper to express the respective mouldings and parts: And the order, exclusive of its ornaments, will be delineated.

As the altitudes of many of the parts are very small, it will not be convenient, if possible, to take from the scale of lines, such small parts alone; therefore it may be best to proceed as in the following example of the *Ionic* order. Plate I.

I. IN the pedestal.

To

To the minutes in the baſe, 42 $\frac{1}{2}$, add ſome even number of minutes, ſuppoſe 30, and the ſum is 72 $\frac{1}{2}$; then compoſe a table, ſuch as the following one, wherein the alt of the plinth is, ſubſtracted out of the No 72 $\frac{1}{2}$; then the torus out of this remainder; then the fillet out of this remainder; then the cyma out of this remainder; then the fillet out of this, and laſtly, the cavetto out of this remainder. Thus,

	Min.
Baſe with 30 minutes - - -	$72\frac{1}{2}$
This leſs by the plinth, 28 $\frac{1}{2}$, remains	44
This leſs by the torus, 4, rem. - -	40
This leſs by the fillet, 0 $\frac{3}{4}$, rem. -	$39\frac{1}{4}$
This leſs by the cyma, 5, rem. - -	$34\frac{1}{4}$
This leſs by the fillet, 0 $\frac{3}{4}$, rem. -	$33\frac{1}{2}$
This leſs by the cavetto 3 $\frac{1}{2}$, rem.	30, the minutes firſt added.

THEN the ſector being fitted to the ſcale intended, and a point A, in the axis, taken to begin at; lay from that point, on the axis, the firſt No 72 $\frac{1}{2}$; this reaches from **A** to **B**, and includes the baſe, and 30 minutes taken in the die. From the point **B**, lay off towards A, the ſeveral numbers expreſs'd in the foregoing tablet; and they will give the reſpective altitudes of the members of the baſe.

IT will be found moſt convenient to lay off the numbers from the greater to the leſſer ones; for then there is only one motion required in the joints of the compaſſes, which is, to bring them cloſer and cloſer every diſtance laid down.

AND in the same manner, for the cornice of the pedestal, take a point C, 30 minutes below the cornice; and tabulate as before.

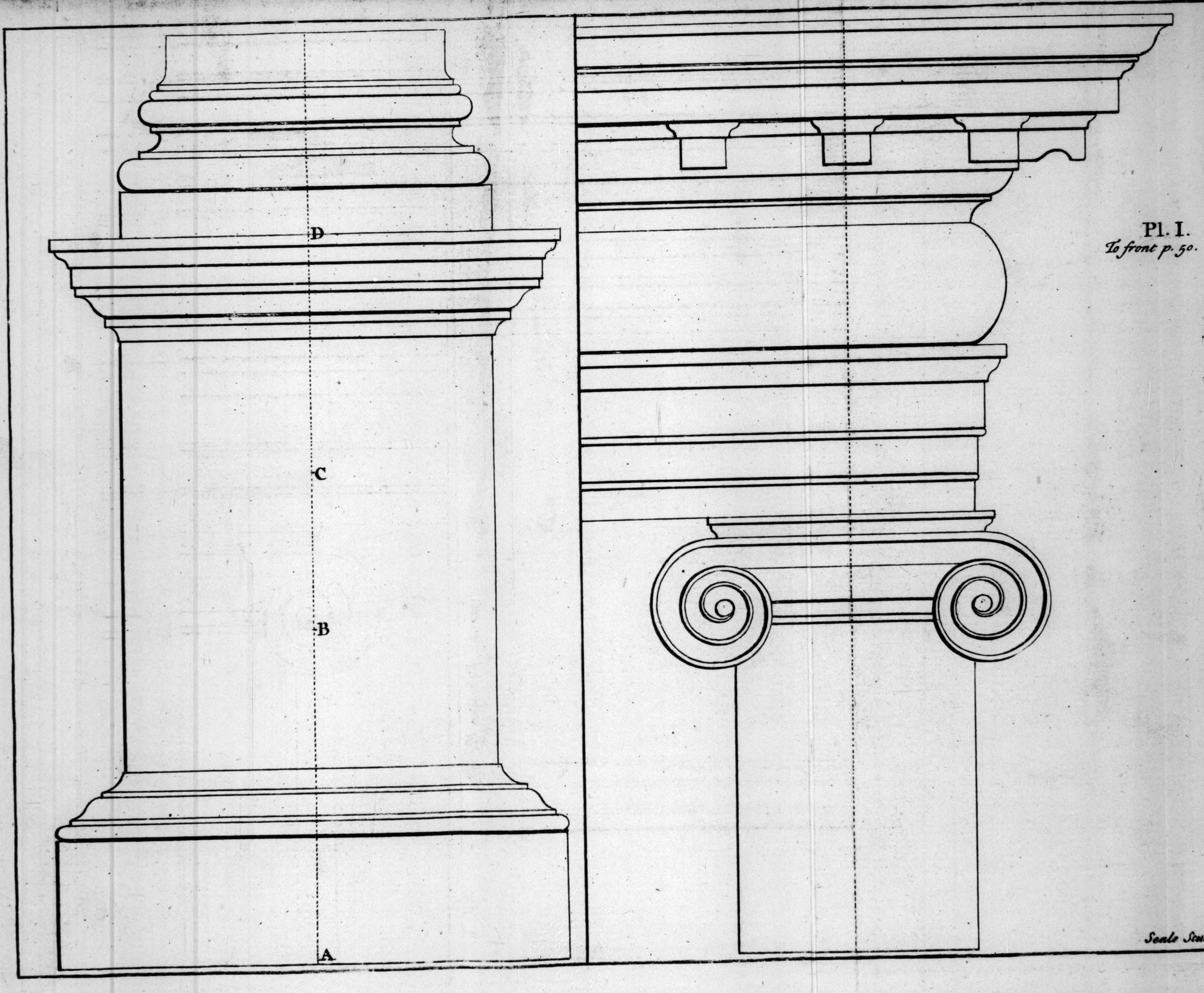
Cornice with 30 min. - - - - - -				$52\frac{3}{4}$
This less by the fillet or cap		$2\frac{1}{2}$	leaves	$50\frac{1}{4}$
Ditto - - - -	ogee - -	$3\frac{1}{2}$,	ditto -	$46\frac{3}{4}$
Ditto - - - - - -	corona	$4\frac{1}{2}$,	- - - -	$42\frac{1}{4}$
Ditto - - - -	fillet -	$1\frac{3}{4}$,	- - -	$40\frac{1}{2}$
Ditto - - - - - -	cyma -	$5\frac{1}{4}$,	- - - -	$35\frac{1}{4}$
Ditto - - - -	fillet -	$1\frac{3}{4}$,	- - - -	$33\frac{1}{2}$
Ditto - - - - - -	cavetto	$3\frac{1}{2}$,	- - - -	30

THESE numbers laid from C to D, gives the altitudes of the members of the cornice.

IN the following tables, 30 min. are added to the altitudes of the base and capital of the column: and in those of the entablature, the altitude only, of the freeze 27, is added to the architrave and cornice.

COLUMN.		ENTABLATURE.	
Base.	Capital.	Architrave.	Cornice.
$63\frac{1}{2}$	$54\frac{1}{4}$	63	73
$53\frac{1}{1}$	$52\frac{1}{2}$	$56\frac{1}{2}$	$70\frac{1}{2}$
46	$49\frac{1}{6}$	$55\frac{1}{4}$	$63\frac{1}{2}$
$44\frac{3}{4}$	$47\frac{5}{6}$	$46\frac{11}{12}$	$62\frac{1}{2}$
$40\frac{1}{12}$	$42\frac{1}{2}$	$44\frac{11}{12}$	59
$38\frac{5}{6}$	35	$34\frac{5}{12}$	51
$33\frac{1}{2}$	$31\frac{2}{3}$	$29\frac{2}{3}$	48
$31\frac{1}{4}$	30	27	$40\frac{1}{2}$
30			39
			33
			32
			27

A



Pl. I.
To front p. 50.

Seale Sculp.

A little reflection will make this very clear, and perhaps more so, than by bestowing more words thereon.

There are some particulars relating to each order, which could not conveniently be introduced in the tables, and are here supplied in the following remarks, Plate II.

I. In the *Tuscan* order. The ovolo, under the Corona, in the Cornice of the entablature, is commonly continued, within the corona, giving it a reverse bend in the sofite, something like unto a cyma, as may be seen in the figure of the order.

II. In the *Doric* order. The second face of the architecture is ornamented with rows of six drips or bells and a plain cap: The freeze, with trigliphs and metops: The breadth of the drips, cap and trigliphs are each 30 minutes; the trigliphs consist of two channels, two half channels, and three voides; the breadth of the channels and voides, are each 5 min. The axis of the column continued, runs through the middle void; leaving the drips, 3 on each side. The metops, or distance between the trigliphs is equal to the height of the freeze, and is commonly ornamented with trophies, arms, roses, &c.

	Alt.	Proj.	Profile.
	Min.	Min.	Min.
Capital	5	16	3
Freeze	45	—	—
Trigliphs	40	15	$\frac{1}{2}+2\frac{1}{2}$
Plinth	$4\frac{1}{2}$	16	3
Cap	$1\frac{2}{3}$	15	2
Drips	$3\frac{1}{3}$	15	2

The column ſign'd profile, ſhews how far the parts project without the planes or faces of the members on which they are made.

The ſofite of the corona in the cornice of the entablature, is frequently ornamented with drips, roſes, &c.

III. In the *Ionic* order. The volutes of the capital are now made to project in the direction of the diagonal of the ſquare cap over the volutes; ſo that their drawing ſhould be expreſs'd like the volutes in the *Roman* order: But theſe are much better drawn by an eaſy hand, than by any rules that can be given, obſerving the limits of their alt. and proj. as in the table of columns.

The freeze is form'd by the ſegment of a circle, whoſe chord paſſes through the lower part of the cavetta of the cornice; and is parallel to the axis.

In the cornice of the entablature, the diſtance of the modillions is 22 min. and the breadth of each 10 min. The axis of the order always paſſes through the middle of a modillion.

IV.

IV. In the *Corinthian* order; the leaves and ſtalks are beſt done by hand, obſerving the altitudes and projections. The bottom of the freeze is commonly turn'd off in a chanfrain, meeting the extremity of the upper fillet of the architrave: The breadth of the dentels are 4 min. and their diſtance 2 min. The diſtance of the modillions is 23 $\frac{1}{4}$ min. and the breadth of each 11 $\frac{1}{3}$ min. the middle of a dentel, is under the middle of each modillion.

V. In the *Roman* order; the capital is form'd from the *Corinthian* and *Ionic*; and the ſame obſervation for the conſtructions of thoſe will ſerve for the conſtructions of this. The freeze is form'd like that of the *Ionic*: The greater diſtance of the modillions, is 23; and the leſſer diſtance is 20: the greater breadth of them is 12 $\frac{1}{2}$ and the leſſer 9 $\frac{1}{2}$.

A TABLE, *shewing the Altitudes and Projections of* *Order; according to the*

Names of the Members.		Tuscan.				Doric.			
		Alt.		Proj.		Alt.		Proj.	
		Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.
CORNICE.	Fillet	-	-	-	-	0	$3\frac{2}{3}$	0	56
	Ogee	-	-	-	-	-	-	-	-
	Corona	-	-	-	-	-	-	-	-
	Fillet	-	-	-	-	-	-	-	-
	Cyma	-	-	-	-	0	9	-	-
	Fillet	-	-	-	-	0	$\begin{cases}1\frac{1}{4}\\1\frac{1}{4}\end{cases}$	0	$\begin{cases}47\\45\frac{3}{4}\end{cases}$
	Astragal	-	-	-	-	-	-	-	-
	Ogee	-	-	-	-	-	-	-	-
	Cavetto	-	-	-	-	0	5	0	$41\frac{1}{4}$
	Fillet	-	-	-	-	-	-	-	-
The Cornice		-	-	-	-	0	$26\frac{1}{6}$	-	-
THE DIE		1	0	0	42	1	20	0	40
The Base		-	-	-	-	0	40	-	-
BASE.	Fillet	-	-	-	-	-	-	-	-
	Cavetto	-	-	-	-	0	5	0	$41\frac{1}{4}$
	Ogee	-	-	-	-	-	-	-	-
	Astragal	-	-	-	-	-	-	-	-
	Fillet	-	-	-	-	0	$\begin{cases}1\frac{1}{4}\\1\frac{1}{4}\end{cases}$	0	$\begin{cases}46\\47\frac{1}{4}\end{cases}$
	Cyma	-	-	-	-	-	-	-	-
	Fillet	-	-	-	-	-	-	-	-
	Torus	-	-	-	-	0	5	0	50
	Plinth	-	-	-	-	0	$27\frac{1}{2}$	0	50

A TABLE *of general*

The Order	9	$44\frac{1}{2}$	-	-	12	$13\frac{1}{6}$	-	-
The Entablature	1	$44\frac{1}{2}$	-	-	1	53	-	-
The Column	7	0	-	-	8	0	-	-
The Pedestal	1	0	-	-	2	$20\frac{1}{6}$	-	-

every Moulding and Part in the Pedestals of each Proportions given by Palladio.

Ionic.				Corinthian.				Roman.			
Alt.		Proj.		Alt.		Proj.		Alt.		Proj.	
Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.
0	2½	0	56¼	0	2½	0	57	0	2½	0	57
0	3½	0	55¼ 53¼	0	3½	0	56 54¼	0	3½	0	56 54¼
0	4½	0	52¼	0	4¼	0	53¼	0	5½	0	53½
0	1¾	0	51¾	- -	- -	- -	- -	0	1	0	52¾
0	5¼	- -	- -	0	4¼	0	49¼ 46	0	8½	- -	- -
0	1¾	0	44¾	0	0¾	0	46	- -	- -	- -	- -
- -	- -	- -	- -	- -	- -	- -	- -	0	3	0	46¼
- -	- -	- -	- -	0	3¾	0	45 43	- -	- -	- -	- -
0	3½	0	41¾	- -	- -	- -	- -	- -	- -	- -	- -
- -	- -	- -	- -	- -	- -	- -	- -	0	1¾	0	44¾
0	22¾	- -	- -	0	19	- -	- -	0	25¾	- -	- -
1	35	0	41¼	1	36	0	42	2	6½	0	42
0	42½	- -	- -	0	38	- -	- -	0	50	- -	- -
- -	- -	- -	- -	- -	- -	- -	- -	0	1	0	45½
0	3½	0	41¾	- -	- -	- -	- -	- -	- -	- -	- -
- -	- -	- -	- -	0	4	0	43 46	- -	- -	- -	- -
- -	- -	- -	- -	- -	- -	- -	- -	0	3	0	47
0	0¾	0	47¼	0	0¾	0	47	- -	- -	- -	- -
0	5	- -	- -	0	5	- -	- -	0	7½	0	45½ 54¾
0	0¾	0	53¾	0	0¾	0	55	0	1	0	54¾
0	4	0	56¼	0	4	0	57	0	4½	0	57
0	28½	0	56¼	0	23½	0	57	0	33	0	57

Proportions for the Order.

13	29¼	- -	- -	13	57	- -	- -	15	22¼	- -	- -
1	49	- -	- -	1	54	- -	- -	2	0	- -	- -
9	0	- -	- -	9	30	- -	- -	10	0	- -	- -
2	40¼	- -	- -	2	33	- -	- -	3	22¼	- -	- -

A TABLE, *shewing the Altitudes and Projections of*
according to the Propor-

	Names of the Members.	Tuscan.				Doric.			
		Alt.		Proj.		Alt.		Proj.	
		Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.
CAPITAL.	Angular Volutes	- -	- -	- -	- -	- -	- -	-	- -
	Abacus: Ovolo	- -	- -	- -	- -	- -	- -	- -	- -
	Abacus: Fillet	- -	- -	- -	- -	0	$1\frac{1}{4}$	0	$38\frac{3}{4}$
	Abacus: Cavetto	- -	- -	- -	- -	- -	- -	- -	- -
	Basket Rim	- -	- -	- -	- -	- -	- -	- -	- -
	Ogee	- -	- -	- -	- -	0	$2\frac{1}{2}$	0	$37\frac{3}{4}$ $36\frac{1}{4}$
	Abacus	0	10	0	30	0	$6\frac{1}{4}$	0	$35\frac{3}{4}$
	Volute: Fillet or Rim	- -	- -	- -	- -	- -	- -	- -	- -
	Volute: Channel or Hollow	- -	- -	- -	- -	- -	- -	- -	- -
	Ovolo	0	10	0	29	0	$6\frac{1}{2}$	0	$34\frac{1}{3}$
	Astragal	- -	- -	- -	- -	- -	- -	- -	-
	Fillet	0	$1\frac{1}{2}$	0	$24\frac{1}{2}$	0 0 0	$1\frac{1}{9}$ $1\frac{1}{9}$ $1\frac{1}{9}$	0 0 0	$29\frac{3}{4}$ $28\frac{1}{2}$ $27\frac{1}{4}$
	Collarino	0	$8\frac{1}{2}$	0	$22\frac{1}{2}$	0	10	0	26
	Middle Volute	- -	- -	- -	- -	- -	- -	- -	- -
	Courses of leaves, folding half their height: 3d	- -	- -	- -	- -	- -	- -	- -	- -
	2d	- -	- -	- -	- -	- -	- -	- -	- -
	1st	- -	- -	- -	- -	- -	- -	- -	- -
SHAFT.	Astragal	0	4	0	27	0	$3\frac{1}{2}$	0	30
	Fillet	0	$1\frac{1}{2}$	0	$24\frac{1}{2}$	0	$1\frac{1}{2}$	0	$28\frac{1}{4}$
	Body of the Column	5	$54\frac{1}{2}$	0	$22\frac{1}{2}$ 30	6	$53\frac{3}{4}$	0	26 30
	Fillet	0	$2\frac{1}{2}$	0	$33\frac{3}{4}$	0	$1\frac{1}{4}$	0	$33\frac{1}{2}$
	Astragal	- -	- -	- -	- -	- -	- -	- -	- -
BASE.	Torus	- -	- -	- -	- -	0	$5\frac{1}{2}$	0	$36\frac{2}{3}$
	Astragal	- -	- -	- -	- -	- -	- -	- -	-
	Fillet	- -	- -	- -	- -	0	$1\frac{1}{4}$	0	35
	Scotia	- -	- -	- -	- -	0	$4\frac{1}{2}$	0	$33\frac{1}{3}$
	Fillet	-	- -	- -	- -	0	$1\frac{1}{4}$	0	$36\frac{2}{3}$
	Astragal	- -	- -	- -	- -	- -	- -	- -	- -
	Fillet	- -	- -	- -	- -	- -	- -	- -	- -
	Scotia	- -	- -	- -	- -	- -	- -	- -	- -
	Fillet	- -	- -	- -	- -	- -	- -	- -	- -
	Torus	0	$12\frac{1}{2}$	0	40	0	$7\frac{1}{2}$	0	40
	Plinth	0	15	0	40	0	10	0	40
	Base	0	$27\frac{1}{2}$	- -	- -	0	30	- -	- -
	Shaft	6	$2\frac{1}{2}$	- -	-	7	0	- -	- -
	Capital	0	30	- -	- -	0	30	- -	- -

SECOND.

every Moulding and Part in the Columns of each Order;
tions given by Palladio.

Ionic.				Corinthian.				Roman.			
Alt.		Proj.		Alt.		Proj.		Alt.		Proj.	
Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.
0	$26\frac{3}{2}$	0	$41\frac{2}{3}$	0	12	0	41	0	$25\frac{2}{3}$	0	35
- -	- -	- -	- -	0	3	0	45	0	3	0	44
0	$1\frac{3}{4}$	0	$31\frac{1}{2}$	0	$1\frac{1}{3}$	0	42	0	$1\frac{1}{3}$	0	$42\frac{1}{2}$
- -	- -	- -	- -	0	$5\frac{2}{3}$	0	39	0	$5\frac{2}{3}$	0	41
- -	- -	- -	- -	0	$2\frac{1}{2}$	- -	- -	- -	- -	- -	- -
0	$3\frac{1}{2}$	0	$30\frac{3}{4}$ 29	- -	- -	- -	- -	- -	- -	- -	- -
- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -
0	$1\frac{1}{3}$	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -
0	$5\frac{1}{3}$	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -
0	$7\frac{1}{2}$	0	35	- -	- -	- -	- -	0	$5\frac{1}{2}$	0	32
- -	- -	- -	- -	- -	- -	- -	- -	0	3	0	26
- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -
- -	- -	- -	- -	- -	- -	- -	- -	0	$1\frac{1}{2}$	0	24
- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -
- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -
- -	- -	- -	- -	0	$9\frac{1}{2}$	- -	- -	- -	- -	- -	- -
- -	- -	- -	- -	0	8	- -	- -	0	10	- -	- -
- -	- -	- -	- -	0	20	0	41	0	20	0	39
- -	- -	- -	- -	0	20	0	35	0	20	0	35
0	$3\frac{1}{3}$	0	30	0	$3\frac{2}{3}$	0	30	0	4	0	30
0	$1\frac{2}{3}$	0	$28\frac{1}{3}$	0	$1\frac{1}{3}$	0	28	0	$1\frac{1}{2}$	0	28
8	$2\frac{1}{4}$	0	26 30	7	$40\frac{3}{4}$	0	26 30	8	9	0	26 30
0	$1\frac{1}{4}$	0	33	0	$1\frac{3}{4}$	0	$33\frac{1}{2}$	0	1	0	34
0	$2\frac{1}{4}$	0	$34\frac{1}{2}$	0	$2\frac{1}{2}$	0	$35\frac{1}{2}$	0	3	0	$35\frac{1}{2}$
0	$5\frac{1}{3}$	0	37	0	5	0	$37\frac{1}{2}$	0	$4\frac{1}{2}$	0	37
- -	- -	- -	- -	0	$1\frac{1}{2}$	0	$35\frac{1}{2}$	- -	- -	- -	- -
0	$1\frac{1}{4}$	0	$34\frac{1}{2}$	0	$0\frac{2}{3}$	0	34	0	c$\frac{2}{3}$	0	$35\frac{1}{2}$
0	$4\frac{2}{3}$	- -	- -	0	$3\frac{3}{4}$	- -	- -	0	3	- -	- -
0	$1\frac{1}{4}$	0	37	0	c$\frac{3}{4}$	0	37	0	$0\frac{1}{2}$	0	$36\frac{1}{2}$
- -	- -	- -	- -	0	$1\frac{3}{4}$	0	$38\frac{1}{2}$	0 0	1 1	0 0	37 37
- -	- -	- -	- -	- -	- -	- -	- -	0	$0\frac{1}{2}$	0	$36\frac{1}{2}$
- -	- -	- -	- -	- -	- -	- -	- -	0	3	- -	- -
- -	- -	- -	- -	- -	- -	- -	- -	0	c$\frac{2}{3}$	0	$38\frac{1}{2}$
0	$7\frac{1}{2}$	0	$41\frac{1}{4}$	0	7	0	42	0	7	0	42
0	10	0	$41\frac{1}{4}$	0	$9\frac{2}{3}$	0	42	0	$9\frac{2}{3}$	0	42
0	30	- -	- -	0	30	- -	- -	0	$31\frac{1}{2}$	- -	- -
8	$10\frac{3}{4}$	- -	- -	7	5	- -	- -	8	$18\frac{1}{2}$	- -	- -
0	$19\frac{1}{4}$	- -	- -	1	10	- -	- -	1	10	- -	- -

A TABLE, *shewing the Altitudes and Projections of* *Order; according to the Pro-*

	Names of the Members.	Tuscan.				Doric.			
		Alt.		Proj.		Alt.		Proj.	
		Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.
CORNICE.	Fillet	0	$3\frac{1}{2}$	1	6	0	$2\frac{1}{4}$	1	16
	Cyma	0	10	- -	- -	0	$6\frac{5}{4}$	- -	- -
	Fillet	0	2	0	$54\frac{1}{4}$	0	$0\frac{3}{4}$	1	8
	Ogee	- -	- -	- -	- -	0	$3\frac{1}{4}$	1 1	7 $5\frac{1}{2}$
	Corona	0	10	0	$52\frac{1}{4}$	0	8	1	$4\frac{1}{2}$
	Ovolo	0	9	0	42	0	6	0	$39\frac{1}{2}$
	Fillet or Astragal	0	$1\frac{1}{2}$	0	39	0	1	0	$35\frac{1}{2}$
	Ogee	- -	- -	- -	- -	- -	- -	- -	- -
	Modillion: Second Face	- -	- -	- -	- -	- -	- -	- -	- -
	Modillion: Ogee	- -	- -	- -	- -	- -	- -	- -	- -
	Modillion: First Face	- -	- -	- -	- -	- -	- -	- -	- -
	Fillet	- -	- -	- -	- -	- -	- -	- -	- -
	Ovolo	- -	- -	- -	- -	- -	- -	- -	- -
	Ogee	- -	- -	- -	- -	- -	- -	- -	- -
	Fillet	- -	- -	- -	- -	- -	- -	- -	- -
	Dentel	- -	- -	- -	- -	- -	- -	- -	- -
	Astragal	- -	- -	- -	- -	- -	- -	- -	- -
	Fillet	- -	- -	- -	- -	- -	- -	- -	- -
	Ogee	- -	- -	- -	- -	- -	- -	- -	- -
	Cavetto	0	$7\frac{1}{2}$	0	$23\frac{1}{2}$	0	5	0	31
	Triglyphs Capital	- -	- -	- -	- -	0	5	0	$30\frac{1}{2}$
	The Cornish	0	$43\frac{1}{2}$	- -	- -	0	38	- -	- -
	The Freeze	0	26	0	$22\frac{1}{2}$	0	45	0	26
	The Architrave	0	35	- -	- -	0	30	- -	- -
ARCHITRAVE.	Fillet	0	5	0	$27\frac{1}{2}$	0	$4\frac{1}{2}$	0	28
	Cavetto	- -	- -	- -	- -	- -	- -	- -	- -
	Ogee	- -	- -	- -	- -	- -	- -	- -	- -
	Astragal or Fusarole	- -	- -	- -	- -	- -	- -	- -	- -
	Third Face	- -	- -	- -	- -	- -	- -	- -	- -
	Astragal or Fusarole	- -	- -	- -	- -	- -	- -	- -	- -
	Second Face	0	$17\frac{1}{2}$	0	24	0	$14\frac{1}{2}$	0	27
	Ogee	- -	- -	- -	- -	- -	- -	- -	- -
	Astragal or Fusarole	- -	- -	- -	- -	- -	- -	- -	- -
	First Face	0	$12\frac{1}{2}$	0	$22\frac{1}{2}$	0	11	0	26

every Moulding and Part in the Entablature of each portions given by Palladio.

Ionic.				Corinthian.				Roman.			
Alt.		Proj.		Alt.		Proj.		Alt.		Proj.	
Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.	Mo.	Mi.
0	2½	1	12	0	2¼	1	14	0	2½	1	18½
0	7	- -	- -	0	6¼	- -	- -	0	8	- -	- -
0	1	1	4	0	0⅔	1	6½	0	1	1	10
0	3½	1 1	3 0½	0	3	1 1	5½ 4	0	3¾	1 1	9 6
0	8	0	59½	0	7⅓	1	3	0	9½	1	5
- -	- -	- -	- -	- -	- -	- -	- -	0	2½	0	55
- -	- -	- -	- -	0	0⅔	1	2	0	1¼	0	54
0	3	0	55 53	0	2⅓	1 0	1 59	- -	- -	- -	- -
	- -	- -	- -	- -	- -	- -	- -	0	6½	0	53
0	7½	0	52	0	7¼	0	40¼	0	1¾	0	52½
	- -	- -	- -	- -	- -	- -	- -	0	3¼	0	51
0	1½	0	37	0	1	0	40	0	1	0	51
0	6	0	36	0	4½	0	39	- -	- -	- -	-
- -	- -	- -	- -	- -	- -	- -	- -	0	5	0	35½ 29
- -	- -	- -	- -	0	1	0	36	- -	- -	- -	- -
- -	- -	- -	- -	0	5½	0	35	- -	- -	- -	- -
- -	- -	- -	- -	- -	- -	- -	- -	0	2	0	30
0	1	0	31½	0	1	0	32	0	2	0	28½
- -	- -	- -	- -	0	4½	0	31 27	- -	- -	- -	- -
0	5	0	27	- -	- -	- -	- -	- -	- -	- -	- -
- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -	- -
0	46	- -	- -	0	47½	- -	- -	0	50	- -	- -
0	27	0	34	0	28½	0	26	0	30	0	35
0	36	- -	- -	0	38	- -	- -	0	40	- -	- -
0	2⅔	0	34	0	2½	0	34½	0	2⅛	0	35
- -	- -	- -	- -	- -	- -	- -	- -	0	4⅛	0	32
0	4¾	0	33 30	0	5	0	33½ 30½	0	3⅔	0	31 29
- -	- -	- -	- -	0	2	0	29½	- -	- -	- -	- -
0	10½	0	29	0	10½	0	28	- -	- -	- -	- -
0	2	0	29	0	1¾	0	28	0	$1\frac{5}{12}$	- -	29
0	8⅓	0	27½	0	8¼	0	27	0	15	0	28
- -	- -	- -	- -	- -	- -	- -	- -	0	2⅔	0	27½ 26½
0	1¼	0	27½	0	1¾	0	27	- -	- -	- -	-
0	6½	0	26½	0	6¼	0	26	0	11	0	26

Sect. XII.

Some uses of the Scales of Polygons.

PROBLEM. XIV.

In a given circle, whose diameter is AB, *to inscribe a regular octagon.* Fig. 22.

Sol. Open the legs of the sector, till the transverse distance of 6 and 6, be equal to AB: Then will the transverse distance of 8 and 8, be the side of an octogon which will be inscrib'd in the given circle.

In like manner may any other polygon not exceeding 12 sides, be inscrib'd in a given circle.

PROBLEM. XV.

On a given line AB, *to describe a regular Pentagon.* Fig. 23.

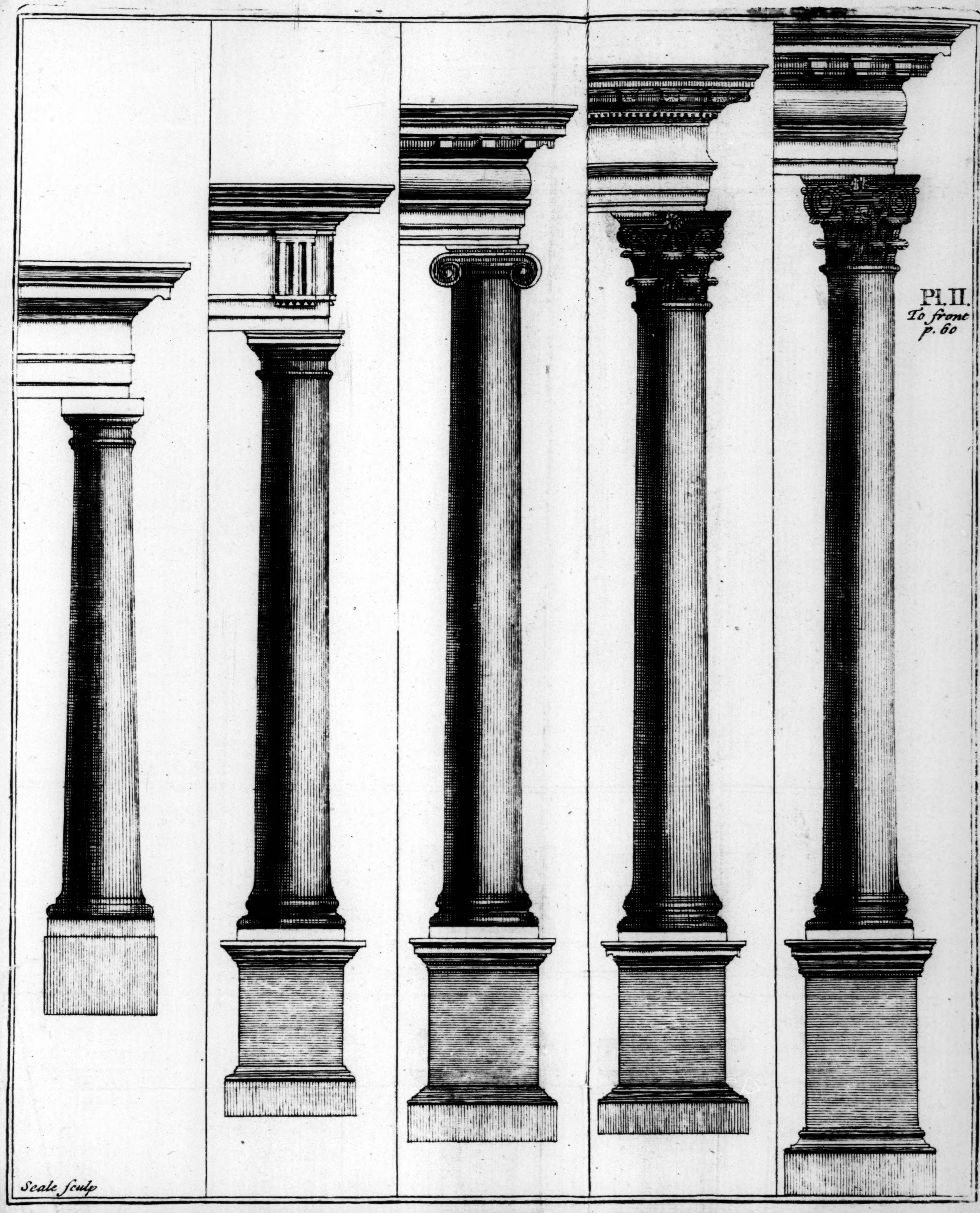
Sol. 1st. Make AB a transverse distance to 5 and 5.

2d. At that opening of the sector, take the transverse distance of 6 and 6; and with this radius, on the points, A, B, as centers, describe arcs cutting in C.

3d. On C as a center, with the same radius, describe a circumference passing through the points A, B: and in this circle may the pentagon, whose side is AB, be inscrib'd.

By a like process may any other polygon, of not more than 12 sides, be described on a given line.

The


Pl. II.
To front p. 60
Seale sculp

THE scales of chords will solve these two problems, or any other of the like kind: Thus,

In a circle whose diameter is AB, *to describe a regular polygon of* 24 *sides.* Fig. 24.

SOL. 1st. Make the diameter AB, a transverse distance to 60 and 60.

2d. Divide 360 by 24; the quotient gives 15.

3d. Take the transverse distance of 15 and 15, and this will be the chord of the 24th part of the circumference.

As there are great difficulties attend the taking of divisions accurately from scales; therefore in this problem, where a distance is to be repeated several times, it will be best to proceed thus.

WITH the chord of 60 degrees, divide the circumference into six equal parts.

IN every division of 60 degrees, lay down 1st. the chord of 15 degrees. 2d. The chord of 30 degrees. 3d. The chord of 45 degrees, beginning always at the same point.

IF methods like this be pursued in all similar cases, the error in taking distances, will not be multiplied into any of the divisions following the first.

SECT. XIII.

Some uses of the scales of chords.

THESE double scales of chords, are more convenient than the single scales, such as described on the plain scale; for on the sector, the

the radius with which the arc is to be deſcrib'd, may be of any length between the tranſverſe diſtance of 60 and 60, when the legs are cloſe, and that of the tranſverſe diſtance of 60 and 60, when the legs are opened as far as the inſtrument will admit off. But with the chords on the plain ſcale, the arc deſcrib'd, muſt be always of the ſame radius.

PROBLEM. XVI.

To protract, or lay down, a right lin'd angle, BAC, *which ſhall contain a given number of degrees.*

CASE I. *When the degrees given are under* 60 : *ſup.* 46. Fig. 25.

1ſt. At any opening of the ſector, take the tranſverſe diſtance of 60 and 60, (on the chords;) and with this opening, deſcribe an arc BC.

2d. Take the tranſverſe diſtance of the given degrees 46, and lay this diſtance on the arc from any point B, to C; marking the extremities B, C, of the ſaid diſtance.

3d. From the center A of the arc, draw two lines AC, BC, each paſſing through one extremity of the diſtance BC, laid on the arc; and theſe two lines will contain the angle required.

CASE II. *When the degrees given are more than* 60. *ſup.* 148°.

1ſt. Deſcribe the arc BC as before.

2d. Take the tranſverſe diſtance of $\frac{1}{2}$ or $\frac{1}{3}$, of the given degrees 148; ſup. $\frac{1}{3} = 49\frac{1}{3}$ degrees

degrees; lay this diſtance on the arc thrice; *viz.* from B to *a*, from *a* to *b*, from *b* to D.

3d. From the center A, draw two lines AB, AD; and the angle BAD will contain the degrees required.

When an angle containing leſs than 5 degrees, ſuppoſe 3 $\frac{1}{2}$, is to be made, 'tis moſt convenient to proceed thus.

1ſt. Deſcribe the arch DG with the chord of 60 degrees.

2d. From ſome point D, lay the chord of 60 degrees to G; and the chord of 56 $\frac{1}{2}$ degrees ($= 60^\circ - 3^\circ \frac{1}{2}$) from D to E.

3d. Lines drrwn from the center A, thro' G and E, will form the angle AGE, of 3 $\frac{1}{2}$ degrees.

If the radius of the arc or circle is to be of a given length; then make the tranſverſe diſtance of 60 and 60, equal to that aſſign'd length.

Either of theſe ſcales of chords, may be uſed ſingly in the manner directed in the uſe of chords on the plane ſcale.

From what has been ſaid about the protracting of an angle to contain a given number of degrees, it will be eaſy to ſee how to find the degrees which are contain'd in a given angle already laid down.

PROBLEM. XVII.

To delineate the viſual lines of a ſurvey; by having given, the bearings and diſtances from each other, of the ſtations terminating thoſe viſual lines.

Ex-

EXAM. Suppose in the field book of a survey, the bearings and distances of the stations were express'd as follows.

⊙ signifies Station.
B ——— Bearing.
D ——— Distance.

	⊙ 1.	B. 289°. 10′.	D. 1080. Links.	
	⊙ 2.	B. 231. 50.	D. 580.	
	⊙ 3.	B. 327. 45.	D. 605.	
	⊙ 4.	B. 72. 30.	D. 766.	
	⊙ 5.	B. 309. 15.	D. 940.	
	⊙ 6.	B. 86. 5.	D. 1085.	
	⊙ 7.	B. 176. 35.	D. 700.	
Return to D. 314 in ⊙ 7.	⊙ 8.	B. 226. 30.	D. 510 to ⊙ 5.	
Return to D. 700 in ⊙ 7.	⊙ 9.	B. 173. 30.	D. 390 to ⊙ 2.	
Return to ⊙ 10.	⊙ 10.	B. 150. 40.	D. 668 cutting 1st D.	
	⊙ 11.	B. 84. 30.	D. 800.	
	⊙ 12.	B. 188. 10.	D. 784 to ⊙ 1.	

THE

THE bearings are counted from the North, Westward. Therefore all bearings under 90 degrees, fall between the N. and W. or in the 1st quadrant.

BEARINGS between 90° and 180°, fall between the W. and S. or in the 2d quadrant.

THOSE between 180° and 270°, fall between the S. and E. or in the 3d quadrant.

AND those between 270° and 360°, fall between the E. and N. or in the 4th quadrant.

SOL. 1st. Take the transverse distance of 60 and 60, (the sector being opened at pleasure,) and with this radius describe a circumference.

2d. Draw the diameters N S. W E. at right angles.

3d. The first bearing being 289° 10′; or 270° 0′, and 19° 10′; take the transverse distance 19° 10′, and lay it on the circumference in the 4th quadrant, from E. towards N.

4th. The 2d bearing is 231° 50′; or 180° 0′ + 51° 50′, and falls in the third quadrant, therefore take the transverse distance of 51° 50′, and lay it from S. towards E. and thus proceed with all the bearings, marking the terminating points in the circumference with the numbers 1, 2, 3, 4, &c. corresponding to the number of its respective bearing

5th. Chuse some point on the paper to begin at, as ⊙ 1.

6th. Lay a parallel ruler by the center of the circle C and No. 1. on the circumference; and parallel thereto, passing through ⊙ 1, draw a line ⊙ 1, ⊙ 2, of a length equal to the first distance, *viz.* D. 108c.

7th. Lay the ruler by the center C and N° 2. and parallel thereto, paſſing through ⊙ 2, draw the line ⊙ 2 ⊙ 3, equal to D. 580.

8. Alſo parallel to C 3, draw the line ⊙ 3 ⊙ 4 = 605.

	C 4,	———— ⊙ 4 ⊙ 5	= 766.
	C 5,	———— ⊙ 5 ⊙ 6	= 940.
	C 6,	———— ⊙ 6 ⊙ 7	= 1085.
	C 7,	———— ⊙ 7 ⊙ 10	= 700.
In the line ⊙ 7 ⊙ 10, take ⊙ 7 ⊙ 8 = 314 &	C 8,	———— ⊙ 8 ⊙ 5	= 510.
	C 9,	———— ⊙ 5 ⊙ 2	= 390.
	C 10,	———— ⊙ 10*a*	= 668.
	C 11,	———— ⊙ 10 ⊙ 12	= 800.
	C 12,	———— ⊙ 12 ⊙ 1	= 784.

And the viſual lines of the ſurvey will be delineated.

SECT.

Sect. XIV.

Some Uses of the Logarithmic Scale of Numbers.

BEFORE any operations can be performed by this ſcale, the notation, or the eſtimating of the values of the ſeveral diviſions, muſt be well known.

Therefore, the Sector being quite opened,

If the 1 at the beginning of the ſcale, or of the 1ſt interval, be taken for	1, 10, 100, &c. $\frac{1}{10}$, $\frac{1}{100}$, &c.
Then the 1 in the middle, or at the end of the 1ſt interval and the beginning of the ſecond, will expreſs	10, 100, 1000, &c. 1, $\frac{1}{10}$, &c.
And the 10 at the end of the 2d interval, or end of the ſcale, will repreſent	100, 1000, 10000, &c. 10, 1, &c.

And

And the primary and intermediate divisions in each interval, must be estimated according to the values set on their extremities, *viz.* at the beginning, middle and end of the scale.

In arithmetical multiplication, or division; the parts may be considered as proportional terms; for in simple multiplication; as unity or 1, is to one factor; so is the other factor, to the product: And in division; as the divisor, is to unity; (or to the dividend,) so is the dividend, (or unity,) to the quotient.

Now as the common logarithms of numbers, express how far the ratios of their corresponding numbers are distant from unity; it follows, that of those numbers which are proportional, that is, have equal ratios; their corresponding logarithms will have equal intervals, or distances: and hence arises the rule for working proportionals on the logarithmic scale.

Rule. Set one point of the compasses on the first term, and extend the other to the second term: Keep the compasses thus opened; set one point on the third term, and the other point will fall on the fourth term, or number sought.

Exam. I. *What is the product of 3 by 4?*

Sol. Set one point on the 1 at the beginning, and extend the other to 3, in the first interval; with this opening, set one point on 4, in the first interval, and the other will reach to 12, found in the second interval.

Observe. In this Exam. the 1, 3, and 4, are valued as units in the first interval; and the

the one in the middle is 10; the distance between this 1 or 10, and the 2 or 20, in the second interval, is divided into 10 principal parts, express'd by the longer strokes; every one in this Exam. is taken as an unit; now as the point of the compasses falls on the second of these principal parts, that is on 2 units beyond 10; therefore this point is to be esteemed in this Exam. as 12.

EXAM. II. *What is the product of* 40 *by* 3?

SOL. In the first interval, take the distance between 1 and 3; and this distance will reach from (4 or) 40 in the first interval to (12 or) 120 in the second interval.

Observe. The 1 and 3 in the first interval, are taken as units; but as the values given to the divisions in either interval, may as well be call'd 40, as 4; and being taken as 40, the 1 at the beginning of the second interval will be 100; and the 2 in the second interval will be 200: consequently the principal divisions between this 1 and 2 will each express 10; and so the second of them will be 20, which with the 100, express'd by the 1, makes 120.

EXAM. III. *What is the product of* 35 *by* 24?

SOL. The distance from 1 in the first interval, to 24 in the second, will reach from 25 in the first interval, to 840 in the second.

Observe. In the first application of the compasses, the primary divisions in the first interval are taken as units, and those in the second interval, as tens: But in the second application, the primary divisions in the first in-

 terval

terval are reckon'd as tens; and thoſe in the ſecond, as hundreds.

As the extent out of one interval into the other, may ſometimes be inconvenient, it will be proper to ſee in ſuch caſes, how the Example may be ſolved in one interval. Thus,

In either interval, take the extent from 1 to $2\frac{4}{10}$ (i. e. 24) and this extent, (in either interval,) will reach from $3\frac{5}{10}$ (i. e. 35:) to $8\frac{4,0}{10,0}$; (i. e. 840.)

In this operation; the ſecond term is reckoned a tenth higher than the firſt term; therefore, as it falls in the ſame interval, the fourth term muſt be a tenth higher than the third term.

Exam. IV. *What is the product of* 375 *by* 60?

Sol. The extent from 1 to 6, (or 60) in the firſt interval will reach from $3\frac{7\frac{1}{2}}{10}$ ($=3\frac{75}{100}$ or 375) in the firſt interval, to $2\frac{25}{100}$ in the ſecond interval; which diviſion muſt be reckoned 22500: For had the point fell in the firſt interval, it would have been one place more than the 375, becauſe 60 is one place more than 1; but as it falls in the ſecond interval, every of whoſe diviſions is one place higher than thoſe in the firſt interval; therefore, it muſt have two places more than 375, which is taken in the firſt interval.

If the operations in theſe examples be well conſidered, it will not be difficult to apply others to the ſcale, and readily to aſſign the value of the reſult.

Exam. V. *What will be the quotient of* 36 *divided by* 4?

Sol. The extent from 4 to 1, in the first interval; will reach from 36 in the second interval to nine in the first.

It is to be observed, that when the second term is greater than the first term; the extents are reckoned from the left hand towards the right: and when the second term is less than the first, the extents are taken from the right hand towards the left: that is, the extents are always counted the same way towards which the terms proceed.

Exam. VI. *If* 144 *be divided by* 9; *what will be the quotient?*

Sol. The extent from 9 to 1, will reach from 144 to 36.

Exam. VII. *If* 1728 *be divided by* 12; *what will be the quotient?*

Sol. The extent from 12 to 1, will reach from 1728 to 144.

Exam. VIII. *To the numbers* 3, 8, 15; *find a 4th proportional.*

Sol. The extent from 3 to 8; will reach from 15 to 40.

Exam. IX. *To the numbers* 5, 12, 38; *find a 4th proportional.*

Sol. The extent from 5 to 12, will reach from 38 to 91 $\frac{1}{5}$.

Exam. X. *To the Numbers* 18, 4, 364; *find a 4th proportional.*

Sol. The extent from 18 to 4; will reach from 364 to 80 $\frac{8}{9}$.

Exam. XI. *To 2 Numbers* 1 *and* 2; *to find a series of continued proportionals.*

Sol. The extent from 1 to 2, will reach from 2 to 4; from 4 to 8 in the first interval; from 8 to 16 in the second interval; from 16 to 32; from 32 to 64; &c. Also the same extent will reach from 1 $\frac{1}{2}$ to 3; from 3 to 6; from 6 to 12; from 12 to 24; from 24 to 48; &c. And the same extent will reach from 2 $\frac{1}{2}$ to 5; from 5 to 10; from 10 to 20; from 20 to 40; &c. And in a like manner proceed, if any other ratio was given besides that of 1 to 2.

This Example is of use, to find if the divisions of the line of numbers, are accurately laid down on the scale.

There are many other uses to which this scale of log. numbers are applicable, and on which several large treatises have been wrote; but the design here, is not to enter into all the uses of the scales on the sector, only to give a few Examples thereof: but after all that has been said, when examples are to be wrought whose result exceeds three places, tis best to do it by the pen, for on instruments, altho' they be very large ones, the answers at best, are but guess'd at.

Sect. XV.

Some uses of the Scales of Log. Sines and Log. Tangents.

The chief uses of these scales, are conjoined with the scale of log. numbers, in the solution of the cases of trigonometry, where there are proportional sides and angles, which

which will be fully exemplified hereafter: But in this place, it will be proper to ſhew, how theſe proportions are applied to the ſcales.

In trigonometrical proportions, there are four terms under conſideration, *viz.* two ſides and two angles commonly; whereof, only three of the terms are given; and the fourth is required. It muſt be remarked, that the ſides in plane trigonometry, are always applied to the ſcale of log. numbers; and the the angles, are either applied to the log. ſines, or to the log. tangents; according as the ſines or tangents are concerned in the proportion. Therefore, when among the three things given, if two of them be ſides, and the other an angle; or if two terms be angles, and the other a ſide.

Rule. On the log. numbers, take the extent of the numbers expreſſing the ſides; and this extent applied from the numbers expreſſing the angle given, will reach to thoſe of the angle required.

Or, the extent of the angles taken, will reach from the ſide given to the ſide required.

SECT. XVI.

Some uses of the double Scales of Sines, Tangents, and Secants.

PROBLEM. XVIII.

Given the radius of a circle (sup. equal to 2 inches) required the sine, and tangent of 28° 30′ *to that radius.*

SOL. Open the sector so that the transverse distance of 90 & 90, on the sines; or of 45 and 45 on the tangents; may be equal to the given radius; *viz.* two inches: Then will the transverse distance of 28° 30′, taken from the sines, be the length of that sine to the given radius; or if taken from the tangents, will be the length of that tangent to the given radius.

But if the secant of 28° 30′ *was required?*

MAKE the given radius two inches, a transverse distance to o and o, at the beginning, of the line of secants; and then take the transverse distance of the degrees wanted, *viz.* 28° 30′.

A Tangent greater than 45 *degrees, (sup.* 60 *degrees) is found thus.*

MAKE the given radius, sup. 2 inches, a transverse distance to 45 and 45 at the beginning of the scale of upper tangents; and then the required degrees 60° 00′ may be taken from this scale.

PRO-

PROBLEM XIX.

Given the length of the ſine, tangent, or ſecant, of any degrees; to find the length of the radius to that ſine, tangent, or ſecant.

Make the given length, a tranſverſe diſtance to its given degrees on its reſpective ſcale : Then

If a ſine	the tranſverſe diſtance of	90 & 90 on the ſines	will be the radius ſought.
If a tangent under 45°		45 & 45 on the tangents	
If a tangent above 45°		45 & 45 on the upper tangents	
If a ſecant		0 & 0 on the ſecants	

PRO-

PROBLEM XX.

To find the length of a versed sine to a given number of degrees, and a given radius.

Make the transverse distance of 90 & 90 on the sines, equal to the given radius.

Take the transverse distance of the sine complement of the given degrees.

If the given degrees are less than 90, substract the sine complement from the radius, leaves the versed sine.

If the given degrees are more than 90, add the sine complement to the radius, gives the versed sine.

PROBLEM XXI.

To open the legs of the sector, so that the corresponding double scales of lines, chords, sines, tangents, may make, each, a right angle.

On the lines, make the lateral distance 10, a distance between 8 on one leg, and 6 on the other leg.

On the sines, make the lateral distance 90, a distance from 45 to 45; or from 40 to 50; or from 30 to 60; or from the sine of any degrees, to their complement.

On the tangents, make the lateral distance of 45, a distance between 30 & 30.

SECT. XVII.

The Use of some of the single and double Scales, applied in the Solution of the Cases of plain Trigonometry.

PROBLEM XXII.

IN any right lin'd plane triangle, any three of the six terms, viz. *sides and angles, (provided one of them be a side) being given, to find the other three.*

This problem consists of three cases.

CASE I. When among the things given, there be a side and its opposite angle.

CASE II. When there is given two sides and the included angle.

CASE III. When the three sides are given.

SOLUTION *of* CASE I.

EXAMPLE I.

In the triangle ABC: Given AB = 56
AC = 64
$\angle$B = 46° 30′
Required $\angle$C, $\angle$A, & BC.

The proportions are as follow, Fig. 26.

As AC : AB :: S, $\angle$ B : S, $\angle$ C. Then 180°, 0′ − $\angle$B + $\angle$C = $\angle$A.

And S. $\angle$B : S $\angle$A :: AC : BC.

First

First by the logarithm scales.

To find the angle C.

The extent from 64 (= AC) to 56 (= AB) on the scales of logarithm numbers, will reach from 46° 30′, to 39° 24′, (= $\angle$ C.) on the scale of logarithm sines.

And $180^\circ\ 0' - (\overline{46^\circ\ 30' + 39^\circ\ 24'} =)$ $85^\circ\ 54' = 94^\circ\ 6' \angle$ A.

To find the Side BC.

The extent from 46° 30′, to the supplement of 94° 6′ on the scale of log. sines, will reach from 64, (= AC) to 88, (= BC) on the scale of logarithm numbers.

Secondly by the double Scales.

To find the Angle C.

1. Take the lateral distance of 64 from the lines.

2. Make this a transverse distance of 46° 30′, on the sines.

3. Take the lateral distance of 56 on the lines.

4. Find the degrees to which this extent is a transverse distance on the sines, *viz.* 39° 24′; and this is the angle sought.

To find the Side BC.

1. Take the lateral distance of 64 from the lines.

2. Make

2. Make this a transverse distance of 46° 30′, on the sines.

3. Take the transverse distance of 85° 54′ (the supplement of 95° 6′) on the sines.

4. Find the lateral distance this extent is equal to, on the lines; and this distance, *viz.* 88, will be the side required.

Ex. II. In the triangle ABC:

Given BC = 74
∠B = 104° 0′
∠C = 28 0
Required AB & BC. Fig. 27.

Now 180° − (104° 0′ + 28° 0′ =) 132° 0′ gives 48° 0′ = ∠A.

The proportions are,
As S, ∠A : BC :: S, ∠C : AB.
And as S, ∠A : BC :: S ∠B : AC.

First by the Logarithm Scales.

To find AB.

The extent from 48° 0′ (= ∠A) to 28° 0′ (= ∠C) on the scale of logarithm sines, will reach from 74 (= BC) to 46,75, (=AB,) on the scale of logarithm numbers.

To find BC.

The extent from 48° 0′ to 76° 0′ (= supplement of, 104° 0′) on the the scale of log. sines, will reach from 74 to 96,6 (=BC) on the scale of logarithm numbers.

Secondly

Secondly by the double Scales.

To find AB.

1. Take the lateral distance 74 on the lines.

2. Make this extent a transverse distance to 48° 0′ on the sines.

3. Take the transverse distance of 28° 0′ on the sines.

4. To this extent find the lateral distance on the lines, *viz.* 46,75 and this will be the length of AB.

To find AC.

1. Take the lateral distance 74 on the lines.

2. Make this extent a transverse distance to 48° 0′ on the sines.

3. Take the transverse distance to the supplement of 104° 0′ on the sines.

4. To this extent, find the lateral distance on the lines, *viz.* 96,6, and this will be the length of AC.

SOLUTION *of* CASE II.

Ex. III. In the triangle ABC:

Given BC = 74
BA = 52
∠B = 68° 0′

Required ∠A; ∠C; & AC. Fig. 28.

The

The proportions are,

Putting $M = (\frac{180^\circ\ 0' - \angle B}{2} =)\ 56^\circ\ 0'$.

As $BC + BA : BC - BA :: t, M : t, N$.

Then $M + N = \angle A$; and $M - N = \angle C$.

As $S, \angle C : BA :: S, \angle B : AC$.

Now $BC + BA = 126$; and $BC - BA = 22$.

First by the Logarithm Scales.

To find the tangent of N.

TAKE the extent from 126 (= sum of the given sides,) to 22 (= diff. of those sides,) on the scale of logarithm numbers; lay this extent from 45° 0′ downwards on the logarithm tangents; stay the lowest point, and bring that which rested on 45 degrees, to 56° 0′. Lay this extent from 45° 0′ downwards, gives 14° 31′ = N.

AND in this manner always proceed, where M is greater than 45° 0′, and N is less.

But when M & N are each $\left\{\begin{matrix}\text{greater}\\ \text{less}\end{matrix}\right\}$ than 45 degrees.

Then the extent from the sum of the sides to their difference, taken on the logarithm numbers, will reach from M $\left\{\begin{matrix}\text{upwards}\\ \text{downwards}\end{matrix}\right\}$ to N. on the log. tangents.

Now because N = 14° 31′: Therefore $\angle A = (56^\circ\ 0' + 14^\circ 31' =)\ 70^\circ\ 31'$.

And $\angle C = (56^\circ\ 0' - 14^\circ\ 31' =)\ 41^\circ\ 29'$.

To find A C.

The extent from $41^\circ\ 29'$ to $68^\circ\ 0'$ on the logarithm sines: Will reach from 52 to 72, 75 = AC on the logarithm numbers.

Secondly by the double Scales.

Because 126 the sum of the sides will be longer than the scales of lines, therefore take 63, the half of 126; and 11, the half of 22, the difference of the sides: for the ratio of 63 to 11, is the same as that of 126 to 22. Then,

1. Take the lateral distance 63 on the scales of lines.
2. Make this extent a transverse distance to 56 degrees, on the upper tangents.
3. Take the transverse distance of 45° on the upper tangents, and make this extent a transverse distance to 45° on the other tangents.
4. Take the lateral distance 11, on the lines.
5. To this extent, find the transverse distance on the tangents, and this will be, $14^\circ\ 31' = N$.

And this is the manner of operation, when M is greater than 45 degrees, and N is less.

But when M & N are each $\left\{\begin{matrix}\text{greater}\\\text{less}\end{matrix}\right\}$ than 45 degrees.

Then the third article of the foregoing operation is omitted.

Now having found the angles A and C, the side A C may be found as in the first or second Examples.

But

But in this caſe, the third ſide AC may be found without knowing the angles. Thus,

1. Take the lateral diſtance of (34 deg.) the half of (68,) the given angle, from the ſines.

2. Make this extent a tranſverſe diſtance to 30 on the ſines.

3. With the ſector thus opened, take the diſtance from 74 on one leg, to 52 on the other leg, each reckon'd on the lines.

4. The lateral diſtance, on the lines, of this extent, gives the ſide A C = 72,75.

From the the two firſt articles of this operation, is learn'd how to ſet the double ſcales to any given angle.

When the included angle B *is* 90 *degrees, the angles* A *and* C *are more readily found as in the following.*

Ex. IV. In the triangle ABC: Fig. 29.

$$\text{Given } AB = 45$$
$$BC = 65$$
$$\angle B = 90$$
$$\text{Required } \angle A;\ \angle C;\ \&\ AC.$$

The proportions are,

For the Angle A.

AB : BC :: Radius : t, $\angle$ A. And 90° − A = $\angle$ B. And AC may be found as directed in the laſt example.

Firſt by the logarithmic Scales.

The extent from 45 to 65, on the numbers: Will reach from 45 degrees to 55° 18′ on the tangents.

Here the angle A is taken equal to $55^\circ\ 18'$, becauſe the ſecond term BC is greater than the firſt term AB: But if the terms were changed, and it was made BC to AB, then the degrees found would be $34^\circ\ 42' = \angle C$.

Secondly by the double Scales.

1. Take the lateral diſtance of the firſt term, from the lines.
2. Make this a tranſverſe diſtance to 45 deg. on the tangents.
3. Take the lateral diſtance of the ſecond term, from the lines.
4. The tranſverſe diſtance of this extent, found on the tangents, gives the degrees in the angle ſought.

If the firſt term is $\left\{\begin{matrix}\text{greater}\\ \text{leſſer}\end{matrix}\right\}$ than the ſecond, then the lateral diſtance of the firſt term, muſt be ſet to 45 degrees on the tangents of the $\left\{\begin{matrix}\text{greater}\\ \text{leſſer}\end{matrix}\right\}$ ſcale; and the lateral diſtance of the ſecond term, muſt be reckon'd on the $\left\{\begin{matrix}\text{greater}\\ \text{leſſer}\end{matrix}\right\}$ tangents.

SOLUTION *of* CASE III. Fig. 30.

In the triangle ABC:

Given BC = 926.
BA = 558.
AC = 702.
Requir'd $\angle$B, $\angle$C. $\angle$A.

Pre-

Preparation to find an Angle, suppose B.

Put A=BC=926 the greater } side including } the angle (B.) sought.
B=BA=558 the lesser
C=AC=702 the side opposite to

$$\left.\begin{array}{l} D=A-B\ \underline{368} \\ E=\tfrac{1}{2}\overline{C+D}=535 \\ F=\tfrac{1}{2}\overline{C-D}=167 \end{array}\right\} \text{Then } 1 : \sqrt{\frac{E\times F}{A\times B}} :: \text{radius} : S, \tfrac{1}{2} \angle B.$$

First by the Logarithmic Scales.

1. The extent from 1 to E = 535, will reach from F = 167 to a 4th point on the log. numbers: Mark the point, and call it G.

2. The extent from 1 to B = 558, will reach from A = 926, to a 4th point on the log. numbers: Mark the point, and call it H.

3. The extent from the point H to the point G, will reach from 1 downward, to a 4th point on the log. numbers: Call this point I.

4. The extent from I, to the middle point between it, and the next 1, above I, on the log. numbers, will reach from 90°, to 24° 34′ on the log. sines.

5. Then 24° 34′ × 2 = 49° 8 = ∠ B.

The other angles may be found by the first Case.

This rule finds the angle B the best, when the work is to be done by the logarithmic

tables: But perhaps the following method will be most esteemed on the logarithmic scales.

Suppose a perpendicular AD (Fig. 30.) drawn to the greatest side BC, from the angle A opposite thereto; this divides the triangle ABC into two right angled triangles BDA, CDA.

Put $A = AC + AB = 1260$.
$B = AC - AB = 144$.

Then the extent from (BC=) 926, to 1260, will reach from 144, to 196, on the scale of log. numbers.

And $\frac{1260 + 196}{2} = 561 = DC$; and

$\frac{1260 - 196}{2} = 365 = BD$.

The extent from (BA=) 558 to (BD=) 365, on the log. numbers, will reach from 90° to $40^\circ\ 52' = (\angle BAD)$ on the log. sines.

Then $90^\circ\ 0' - 40\ 52'$ gives $49^\circ\ 8$ for the $\angle B$.

And the extent from (AB=) 702, to (DC=) 561, on the log. numbers, will reach from 90° to $53^\circ\ 4' = (\angle CAD)$ on the sines.

And $90^\circ\ 0' - 53^\circ\ 4' = 36^\circ\ 56 = \angle C$.

And $(\angle BAD =)\ 40^\circ\ 52' + (\angle CAD =)\ 53^\circ\ 4' = BAC = 93^\circ\ 56'$.

Secondly

Secondly by the double Scales.

To find the Angle B.

1. Take the lateral diſtance 702, (= AC the ſide oppoſite to ∠ B,) from the lines.

2. Open the legs of the ſector, untill this extent will reach from 926 on one ſcale of lines, to 558 on the other ſcale of lines.

3. The tranſverſe diſtance between 30 degrees on the ſcale of ſines, meaſured laterally on the ſines, gives 24° 34, for half the angle B.

The other angles may be found as B was, or according to the directions in ſome of the preceding examples.

Although in theſe examples, oblique triangles were taken as being the moſt general, yet it may be readily ſeen, that examples in right angles are only particular caſes, and may be as eaſily ſolved.

Variety of others might be given in ſurveying, navigation, &c. but theſe would all be reduced to examples like to the foregoing ones; therefore ſuch are omitted to make room for others, leſs common and more curious.

Sect. XVIII.

The Construction of the several Cases of Spherical Triangles by the Scales on the Sector.

THE cases of spherical triangles are six.

Case I. Given two sides, and an angle opposite to one of them.

Case II. Given two angles, and a side opposite to one of them.

Case III. Given two sides, and the included angle.

Case IV. Given two angles, and the included side.

Case. V. Given the three sides.

Case VI. Given the three angles.

These six cases include all the variety that can arise in spherical triangles.

Methods of constructing these cases, were communicated to the author several years ago, by that excellent Mathematician *William Jones*, Esq.

In the following solutions, are given three constructions to every case, whereby each side is laid on the plane of projection, or (as it is commonly called, the) primitive circle.

To abbreviate the directions given in the following constructions, it is to be understood, that the primitive circle is always first described, and two diameters drawn at right angles.

So-

Solution *of* CASE I.

Exam. In the spherical triangle ABD.
Given AB = 29° 50′
DB = 63 59
∠D = 25 55
Required the triangle.

I. *To put* DB *on the primitive circle.* Fig. 1. 1. Pl. VI.

1st. Make DB = chord of 63° 59′, and draw the diameter BE.

2d. From D, with the secant of the ∠D, 25° 55′, cut the diameter ⊙ I in C: on C as a center, with that radius, describe the circumference DA. and the angle BDA will be 25° 55′.

3d. Make B *d* equal to AB, with the chord of 29° 50′.

4th. With the tangent of AB, 29° 30′, from *d*, cut ⊙ B produced in *b*; and from *b*, with that radius, cut DA in A or *a*.

5th. Through B, A, E, describe a circumference, and the triangle DAB will be that required; whose parts DA, ∠B, and ∠A may be thus measured.

6th. Make ⊙ P equal to the tangent of half the angle BDA. *viz.* 12° 57°; then a ruler on P and A, gives *e*; and D *e* measur'd on the chords, gives the degrees in DA, *viz.* 42° 9′.

7th. Draw the diam. F G ⊥ to BE, cutting the circumference BAE in *s*; A ruler by B & *s* gives *f*; make *fg* equal to the chord of

of 90 deg. a ruler on *g* and B, gives *p* in the diameter FG. Then E *g* on the chords gives the angle B = 36° 9′.

8th. A ruler on A and P, gives *n*; and on A and *p*, gives *m*; and *nm* meaſured on the chords, gives 52° 9′, for the ſupplement of the angle DAB, which is 127° 51′.

II. *To put* DA *on the primitive circle*, Fig. 2. 1.

1ſt. With the ſecant of the angle D, 25,° 55′, from D, cut the diameter in C; and on C, with the ſame radius, deſcribe the arc DB.

2d. Make ⊙ P, equal to the tangent of half the angle D; *viz.* 12° 57′ $\frac{1}{2}$.

3d. On the primitive circle, make D *d* equal to the given ſide DB, with the chord of 63° 59′.

4th. A rnler on P and *d*, gives B; then will DB = 63° 59′.

5th. Draw ⊙ B *r*, cutting the primitive circle in *r*.

6th. Make *r x* = the chord of 90°; or twice the chord of 45°.

7th. A ruler on *x* and B, gives *m* on the primitive circle.

8th. Make *m q* = *mp* = chord of 29° 50′.

9th. A right line through *x* & *p*, *x* & *q*, gives *f* & *e* in ⊙ *r*.

10th. On *fe* as a diameter, deſcribe a circumference cutting the primitive circle in A, *a*.

11th. A ruler on A & ⊙, gives F.

12th. Trough A, B, F, deſcribe a circumference, and the triangle ABD is conſtructed with DA on the primitive circle as required.

III.

III. *To put* AB *on the primitive circle.* Fig. 3. 1.

1st. Make AB = the chord of 29° 50′; and draw the diameter BF.

2. In A *b* drawn ⊥ to AG, take A *b* = sine of AB 29° 50′.

3d. Make the angle *b* A *g* = ∠ D, 25° 55′; from A draw A *e* ⊥ to A *g*, and from *d*, the middle of A *b*, draw *de* ⊥ A *b* cutting A *e*, in *e*; from *e*, with *e* A, describe a circumference A *f b*.

4th. From *b*, with the sine of BD, 63° 59′, cut the circumference A*fb* in *f*; and draw A *f*.

5th. From A, draw AC ⊥ to *f* A, meeting E ⊙ (⊥ A ⊙,) continued, in C; and on C, with the radius CA, describe a circumference ADG.

6th. Make B *m* = BD, with the chord of 63° 59′; from *m*, with the tangent of 63° 59′ cut ⊙ B produced, in *n*; on *n*, with the same radius, cut ADG in D.

7th. Through B, D, F, describe a circumference, and the triangle ABD will be that which was required.

SOLUTION *of* CASE II.

EXAM. In the spherical triangle ABD.

Given AD = 42° 9′

∠A = 127 50

∠B = 36 8

Required the triangle:

I. *To put* DB *on the primitive circle.* Fig. 1. 2.

1st

1st. From B, with the secant of ∠ B, 36° 8′, cut the diameter ⊙ E in C; on C, with the same radius, describe the circumference B*a*F, and the angle DBF = the given ∠ B.

2d. Make the angle *naq* = (127° 50′ − 90′ =) 37° 50′.

3d. Make *aq* = tangent of DA, 42° 9′; on ⊙ with the secant of 42° 9′ describe an arc *q* Q: on C with C *q*, cut the arc *q* Q in Q.

4th. Draw Q ⊙ G cutting the primitive circle in D, and BD will be a side of the triangle.

5th. From Q with Q *a*, cut B *a* F in A; and through D, A, G, describe a circumf. and the triangle BAD is that required. Whose parts BD, BA and ∠ D are thus measured.

6th. BD measured on the chords, gives 64 degrees.

7th. Make ⊙P = tangent of half ∠ B, *viz.* 18° 4′; a ruler on P and A gives *x*; then B*x* measured on the chords gives 29° 50′, for BA.

8th. Draw a diameter perpendicular to GD, cutting the circumf. DAG in *s*; a ruler on D and *s* gives *m*; make *mn* 90 degrees, then G *n* measured on the chords, gives 25° 55′ for the ∠ D.

II. *To put* A B *on the primitive circle.* Fig. 2. 2.

1st. From A, with the secant of the supplement of the ∠ A, *viz.* 52° 10′, cut the diameter ⊙F continued in C; on C, with the same radius, describe a circumf. A *a* E.

2d.

2d. Make ʘ P = the tangent of half the suplement of ∠A, *viz.* 26° 5′; and make A *x* = chord of AD, 42° 9′ : a ruler on P and *x*, gives D ; then is A D equal to 42° 9′.

3d. On ʘ, with the tangent of the angle B, 36° 8′, describe an arc *mc*; on D, with the secant of ∠ B, 36° 8′, cut the arc *mc* in *c*; on *c*, with the same radius, describe a circumf. DB, then the triangle ADB, will be that required.

III. *To put* DA *on the primitive circle.* Fig. 2. 3.

1st. Lay down AD with the chord of 42° 9′ : Draw the diameter D F ; and another ʘ H, perpendicular to DF.

2d. On A, with the secant of the sup. of ∠ A, *viz.* 52° 10′, cut the diameter E ʘ in C ; and on C, with the same radius, describe the circumf. ABG.

3d. Make ʘ P equal to the tangent of half the suplement ∠ A, *viz.* 26° 5′, a ruler by G and *p* gives *x*.

4th. Make *xm* = *xn* with chord of ∠ B, 36° 8′ ; a ruler by G and *n*, G and *m*, gives *r*, *s* ; on *b* the middle of *rs*, with the radius *bs*, cut ʘ H in *p*.

5th. A ruler on F and P, gives *h* ; make *h k* = *h* D ; a ruler or F and *k* gives *c* ; with the radius *c* D, describe the circumference DBF ; and the triangle ABD, is that sought.

SOL.

Solution *of* CASE III.

Ex. In the spherical triangle ABD.

Given AB = 29° 50′

BC = 63 59

∢B = 36 8

Required the triangle.

I. *To put* AB *on the primitive circle.* Fig. 1. 3.

1st. Make AB = chord of 29° 50′, draw the diameter BF, and another ⊙ E perpendicular thereto.

2d. From B, with the secant of ∠ B, 36° 8′ cut ⊙ E in C, the center of BDF.

3d. From ⊙, with the tangent of half ∢ B, *viz.* 18° 4′, cut ⊙ E in P, the pole of BDF.

4th. Make B *x* = BD, 63° 50′; a ruler on P and *x*, gives D. Through A, D, G, describe a circumference, and the triangle ADB is that required, whose parts AD, ∢ A, and ∢ C may be thus measured.

5th. A ruler on A and *s* gives *z*, make *z y* = chord of 90° ; a ruler on A and *y* gives *p* the pole of A *s* G ; a ruler on *p* and D, gives *n*, and A *n* measured on the chords gives 42° 8′ for AD.

6th. G *y* measured on the chords, gives 52° 11′ for the supplement of ∢ A ; therefore ∢ A = 127° 49′.

7th. A rular on D and *p*, D and P, gives *r*, *m* ; and *rm*, measured on the chords gives 25° 56′ for the angle BDA.

II. *To*

II. *To put* DB *on the primitive circle.* Fig. 2. 3.

1st. Make DB = chord of 63° 59: draw the diameter **BF** and perpendicular thereto, the diameter ⊙ G.

2d. From B, with the secant of ∠ B, 36° 8′, cut ⊙ G in C; on C with CB, describe the circumference **BAF**.

3d. Make ⊙ P = tangent of half ∠ B, 18° 4′, and **D** *x* = chord of **AB, 29° 50′**, a ruler on P and *x* gives **A**; through D, A, E, describe a circumference, and the triangle **ABD** is that required.

III. *To put* A D *on the primitive circle.* Fig. 3. 3.

1. In a right line *e d*, touching the primitive circle in any point *b*, take *b d* = tangent of BD, 63° 59′; and *b e* = tangent of **AB, 29° 50′**.

2. Make the angle *d b a* = ∠ **B**, 36° 8′, and make *b a* = *be*.

3. From *d*, ⊙, with D *a*, ⊙ *e*, describe arcs crossing in *x*; from *x*, *d*, draw the diameters AE, DF; and others OG, OH, perpendicular to AE,DF.

4. From *d*, *x*, with *d b*, *e b*, describe arcs crossing in B; and draw *d* B, *x* B.

5. From B draw **B**C, perpendicular to *x* **B**, and meeting ⊙ G producèd in C; also draw **B** *c* perpendicular to *d* **B**, and meeting ⊙ H in *c*; then C is the center of a circumference thro' A, B, E; and *c* the center of that

that thro' D, B, F; and the triangle **ABD** is that required.

Solution *of* CASE IV.

Ex. In the spherical triangle **ABD**:
Given $\angle$D = 25° 55′.
$\angle$B = 36° 08′.
DB = 63° 59′.
Required, The triangle.

I. *To put* DB *on the primitive circle.* Fig. 1. 4.

1. Make DB = chord of 63° 59′; draw the diameter BF, and draw ⊙G perpendicular to BF.

2. From B, with the secant of $\angle$B, 36° 8′, cut ⊙G in C; and C will be the center of BAF.

3. From D, with the secant of $\angle$D, 25° 55′; cut ⊙H in *c*, and *c* will be the center of DAE; and the triangle DAB is that which was required; whose parts DA, BA, and $\angle$A, are thus measured.

4. Make ⊙*p* = tangent of $\frac{1}{2}$ $\angle$D, 12° 57′, a rular on *p* and A gives *x*; then D*x* measured on the chords gives 42° 10′ for AD.

5. Make ⊙P = tangent of $\frac{1}{2}$ $\angle$B 18° 4′, a ruler on P and A, gives *z*; then B*z* measured on the chords, gives 29° 54′ for AD.

6. A ruler on A and *p*, A and P, gives *n*, *m*; and *nm* measured on the chords gives 51° 58′ the suplement of the angle A. Therefore $\angle$A = 128° 2′.

II. *To*

II. *To put* DA *on the primitive circle.* Fig. 2. 4.

1st. From D, with the secant of ∠ D, 25° 55′; cut ⊙F in C; and C is the center of the circumference DBE.

2d. Make ⊙ P = tangent of $\frac{1}{2}$ ∠ D, 12° 57′; and make D*x* = chord of BD, 63° 59′; a ruler on P, *x*, gives B; and D B is 63° 59′.

3d. Make the angle CB*c* = ∠ B, 36° 8′; through C, draw *mc* perpendicular to B ⊙, cutting B*c* in *c*; on *c*, with the radius *c* B, describe the circumf. ABG; and the triangle ABD, is that which was required.

III. *To put* AB *on the primitive circle.* Fig. 3. 4.

1st. From B, with the secant of ∠ B, 36° 8′ cut ⊙ F in C; and C is the center of the circumference of B D E.

2. Make B*x* = chord of BD, 63° 59′; and ⊙P = tangent of $\frac{1}{2}$ ∠ B, 18° 4′; a ruler on P and *x* gives D; then is BD = 63° 59′.

3d. Make the angle CD*c* = ∠ D, 25° 55′; then *mc* drawn perpendicular to ⊙ D, meeting D *c* in *c*, gives *c* the center of the cumf. A D G; and the triangle ABD will be that required.

SOLUTION *of* CASE V.

Ex. In the spherical triangle ABD.
Given AB = 29° 50′
AD = 42 9
BD = 63 59
Required, The triangle.

I. *To put* AB *on the primitive circle.* Fig. 1. 2.

1st. Make AB = chord of 29° 50′; draw the diameter BF.

2d. Make A *n* = chord of AD, 42° 9′; and B*m* = chord of BD, 63° 59′.

3d. From *n*, with the tangent of AD, 42° 9′, cut EA produced in C; and from C, with that radius, describe the arc *n n*; from *m*, with the tangent of BD, 63° 59′, cut FB produced in *c*; and from *c*, with the same radius, cut the arc *nn* in D.

4th. Through A, D, E; B, D, F, describe circumferences, and the triangle ADB is that which was required; whose angles A, B, D, are thus measured.

5th. A ruler on A and *a*, gives *x*; on B and *b*, gives *z*; make *xy*, *zv*, each 90°; a ruler on A and *y* gives P, in a perpendicular to AE; and a ruler on B and *v* gives *p*, in a perpendicular to BF.

6th. E*y* measured on the chords, gives 52° 12′ for the supplement of the ∠ A; therefore ∠ A = 127° 48′.

7th. F*v* measured on the chords, gives 36° 10′ for the angle B.

8th.

8th. A ruler on D and P, D and *p*, gives *t* and *s*; and *t s* measured on the chords, gives $25^\circ\ 58'$ for the angle D.

The sides AD, DB, are put on the primitive circle, by a construction so like the foregoing one, that it is needless to repeat it. See figures 2. 5. 3. 5.

SOLUTION *of* CASE VI.

Ex. In the spherical triangle ABD:

Given $\angle A = 127^\circ\ 15'$.
$\angle B = 36^\circ\ 8'$.
$\angle D = 25^\circ\ 55'$.

Required, The triangle.

I. *To put* AB *on the primitive circle.* Fig. 1. 6.

1st. From B, with the secant of $\angle B$, $36^\circ\ 8'$, cut ⊙ F in C, and C will be the center of the circumference through B,D,E.

2. From ⊙, with the tangent of the supplement of $\angle$ A, $52^\circ\ 10'$, describe an arc *xc*.

3d. Make an angle C $aq = \angle D$, $25^\circ\ 55'$; make $aq =$ Bx. (= secant of $52^\circ\ 10'$.)

4th. From C, with the radius C*q*, cut *xc* in *c*; From *c*, with the radius *q a*, describe a circumference ADG; and the triangle ABD, is that which was required: whose sides AB, BD, DA, are measured as follows.

5th. A ruler on B and *a*, A and *b*, gives *d* and *f*; make *d g*, *f h*, each 90 degrees; a ruler on *g* and B, *h* and A, gives P and *p*,

in ⊙ F, ⊙ H, drawn perpendicular to B E, A G.

6th. A ruler on P and D, *p* and D, gives *n* and *m*.

7th. Then BA, B*n*, A*m*, measured on the chords, gives $29^\circ\ 50'$; $63^\circ\ 59$; $42^\circ\ 9'$; for the respective measures of BA, BD, AD.

The directions for this construction, may be easily applied to the putting either of the other sides on the primitive circle. Fig. 2. 6. 3. 6.

Sect. XIX.

Of the proportional Compasses.

THOSE compasses are called proportional, whose joint lies between the points terminating each leg; in such a manner, that when the compasses are opened, the legs form a cross.

Such compasses are either simple or compound.

Simple proportional compasses, are such, whose center is fix'd: One pair of these, serve only for one proportion.

Thus, if a right line is to be divided into 2, 3, 4, 5, &c. equal parts; or the chord of $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, &c. part of a circumference is to be taken; there must be as many of such compasses, as there are distinct operations to be performed.

In each case, take the length of the right line, or of the radius of the circle, between the longer

longer points of the legs; and the diſtance of the ſhorter points will be the part required.

By the longer points, is meant thoſe points which are to be applied to the given line.

Compound proportional compaſſes, are thoſe which center is moveable; whereby, one pair of theſe will perform the office of ſeveral pairs of the ſimple ſort.

In the ſhanks of theſe compaſſes are grooves, wherein ſlides the center, which is made faſt by a nut and ſcrew.

On each ſide of theſe grooves, ſcales are placed; which may be of various ſorts, according to the fancy of the buyer: But the ſcales which the inſtrument-makers commonly put on theſe compaſſes, are only two, *viz.* lines and circles.

By the ſcale of lines, a right line may be divided into a number of equal parts, not exceeding the greateſt number on the ſcale; which is generally 12.

Exam. I. To divide a given right line, (ſuppoſe of $7\frac{1}{2}$ inches long,) into a propos'd number of equal parts. (as 11.)

Operation. Shut the compaſſes; unſcrew the button; move the ſlider until the line acroſs it, coincides with the 11th diviſion on the ſcale of lines; ſcrew the button faſt; open the compaſſes, until the given line can be received between the longer points of the legs; then will the diſtance of the ſhorter points be the 11th part of the given line, as required.

By the ſcale of circles, a regular polygon may be inſcribed in a given circle; provided the

the number of ſides in the polygon, do not exceed the numbers on the ſcale, which commonly proceed to 24.

EXAM. II. To inſcribe in a circle, whoſe radius is known, (ſup. 6 inches) a regular polygon of 12 ſides?

OPERATION. Shut the compaſſes; unſcrew the button; ſlide the center until its mark coincide with the 12th diviſion on the ſcale of circles; ſcrew the button faſt; take the given radius between the longer points of the legs; then will the diſtance of the ſhorter points, be the ſide of the polygon required.

THESE ſcales are applicable to ſeveral other uſes beſide the foregoing ones, in the ſame manner, as the like lines on the ſector are.

FROM theſe operations 'tis evident, that the lengths of the longer and ſhorter legs, (reckoned from the center,) muſt always be proportional to the diſtance of their extremities.

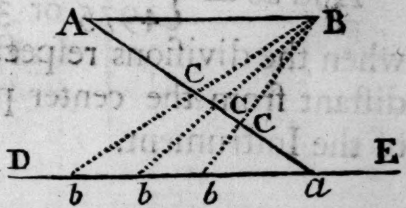
THEREFORE, to divide a right line into 2, 3, 4, 5, 6, 7, 8, &c. equal parts; the lengths of each leg, from the center, will be expreſs'd by the following ſeries, the whole length of the inſtrument being taken for unity.

Longer leg $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}$, &c.
Shorter leg $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}$, &c.

THESE diviſions may be very accurately laid on the legs of the compaſſes by the help of a good ſector. (See Prob. 12.)

OR,

Or, the divisions of this scale of lines may be found by the following construction, which many years ago, was contrived by *William Jones*, Esq; who lately favoured me with a copy thereof, and is thus.



Draw the indefinite right line D E; and from any point A, without D E, draw A*a*, equal to the shank of the compasses, making any angle at *a*, with DE. Through A, draw the right line AB, parallel to DE, and equal to the given line from whose parts the proportions are taken.

Let A*a* contain N parts.

Now that *ab* may be the *nth* part of AB, or, that AB may be *n* times *ab*.

Let $ac = \frac{1}{n+1}$ N, or $Ac = \frac{n}{n+1}$ N;

then the point c is the center of the screw pin. And through c, drawing Bc, meeting DE in *b*; then is $ab = \frac{1}{n}$ of AB, or AB = *n* times *a b*.

For $\frac{ab}{AB} = \frac{ac}{Ac} = \frac{1}{n}$.

If the center of the screw-pin be distant from the mark in the slider, the $\frac{1}{m}$ part of N.

Then $ac = \frac{m \pm s}{s} \times \frac{N}{m}$ (putting $s = n+1$.)

Ex.

Ex. If $N = 10000$, $m = 400$, and $n = 1$, or 2, or 3, *&c.*

Then $ac = 5000$, or 3333, or 2500, *&c.* when the divisions on the shank respect the center pin.

$$\text{And } ac = \begin{cases} 5025 \text{ or } 3358 \text{ or } 2525, \&c. \\ 4975 \text{ or } 3308 \text{ or } 2475, \&c. \end{cases}$$

when the divisions respect a mark in the slider, distant from the center pin, $\frac{1}{25}$ of the length of the Instrument.

THE scale for dividing of the circle, or the divisions for regular polygons may be found thus.

FIND the angles at the center, of as many regular polygons as are to be described on the compasses.

SEEK the sines belonging to the half of each angle, to the radius 1.

TO each of these sines doubled, add the radius 1.

THEN will the reciprocal of these numbers, be the lengths of the polygonal divisions, on the legs of the compasses, reckoned from the longer point; the length of the instrument being accounted unity.

FOR the longer and shorter legs, (or points) are in the same ratio, as are the radius and chord of the angle at the center.

AND as the sum of the radius and chord, is to the radius; so is the sum of the longer and shorter legs, (or points) to the length of the longer point.

AND

And hence was the following table composed, which shews the decimal parts on the leg, from the longer point to the center.

N° Sides.	Length on the Leg.	N° Sides.	Length on the Leg.	N° Sides.	Length on the Leg.
3	0,3333	11	0,6396.	19	0,7523
4	0,4142	12	0,6589	20	0,7617
5	0,4597	13	0,6763	21	0,7706
6	0,5000	14	0,6921	22	0,7785
7	0,5354	15	0,7063	23	0,7860
8	0,5665	16	0,7193	24	0,7931
9	0,5940	17	0,7313		
10	0,6180	18	0,7423		

These divisions may be truely laid off by the help of a good sector; making the whole length of the proportional compasses, a transverse distance to 10 and 10, on the line of lines.

The complements, to unity, of the numbers in the table, will give the distances of the divisions from the other point of the instrument.

If the mark in the slider, is at some distance from the center, as it commonly is, then this distance, which is always known, must be added to, or subtracted from, the foregoing numbers, according to that side of the center the mark is on; and the sums, or remainders, will give the distances of the divisions from one of the points.

About *Michaelmas*, 1746, was finished a pair of proportional compasses, with the addition of a very curious and useful contrivance; (see the plate fronting the title page) *viz.* into one of the legs (A) at a small distance from the end of the groove, was screwed a little pillar (*a*) of about $\frac{1}{3}$ of an inch high, and perpendicular to the said leg; thro' this pillar, and parallel to the leg, went a screw pin; (*bb*) to one end of this screw, was soldered a small beam (*cc*) nearly of the length of the grove in the compasses; the beam was slit down the middle lengthwise, which received a nut (*f*) that slid along the slit; (*dd*) this nut could be screw'd to the beam, fast enough to prevent sliding; one end (*e*) of the screw of the nut *f*, falls into a hole in the bottom of the screw to the great nut (*g*) of the compasses; the screw pin *bb*, passed thro' an adjuster; (*h*) To use this instrument, shut the legs close, slacken the screws of the nuts *g* and *f*; move the nuts and slider *k*, to the division wanted, as near as can be readily done by the hand; screw fast the nut *f*; then by turning the adjuster *h*, the mark on the slider *k*, may be brought exactly to the division; screw fast the nut *g*; open the compasses; gently lift the end *e*, of the screw of the nut *f*, out of the hole in the bottom of the nut *g*; move the beam round its pillar *a*, and slip the point *e*, into the hole in the pin *n*; slacken the screw of the nut *f*; take the given line between the longer points of the compasses, and screw fast the nut *f*: Then may the

the ſhorter points of the compaſſes be uſed without any danger of the legs changing their poſition; this being one of the inconveniencies that attended the proportional compaſſes before this ingenious contrivance; which was made by Mr. *Thomas Heath*, Mathematical Inſtrument-maker, in the *Strand*, *London*; whoſe ſkill in contrivance, and care in executing the workmanſhip of curious inſtruments, is not, perhaps, to be ſurpaſſed by any artiſt.

Soon after the appearance of the firſt proportional compaſſes, there were ſeveral learned and ingenious perſons who contrived a great variety of ſcales to be put thereon; but theſe are here omitted, becauſe the credit of the proportional compaſſes is greatly fallen, ſince the invention of the ſector, the latter being a much more uſeful inſtrument than the former, and not ſo ſubject to be put out of order; for if one of the points of theſe compaſſes ſhould be blunted or broke, the inſtrument cannot be uſed, until the damaged point be replaced by a new one. However, thoſe who are deſirous of knowing the conſtruction and uſe of ſuch ſcales on the proportional compaſſes, may be amply ſatisfied in conſulting *Hulſius*, *Horſcher*, *Galgemaire*, *Bion*, and others mentioned in the preface to this book.

F I N I S.

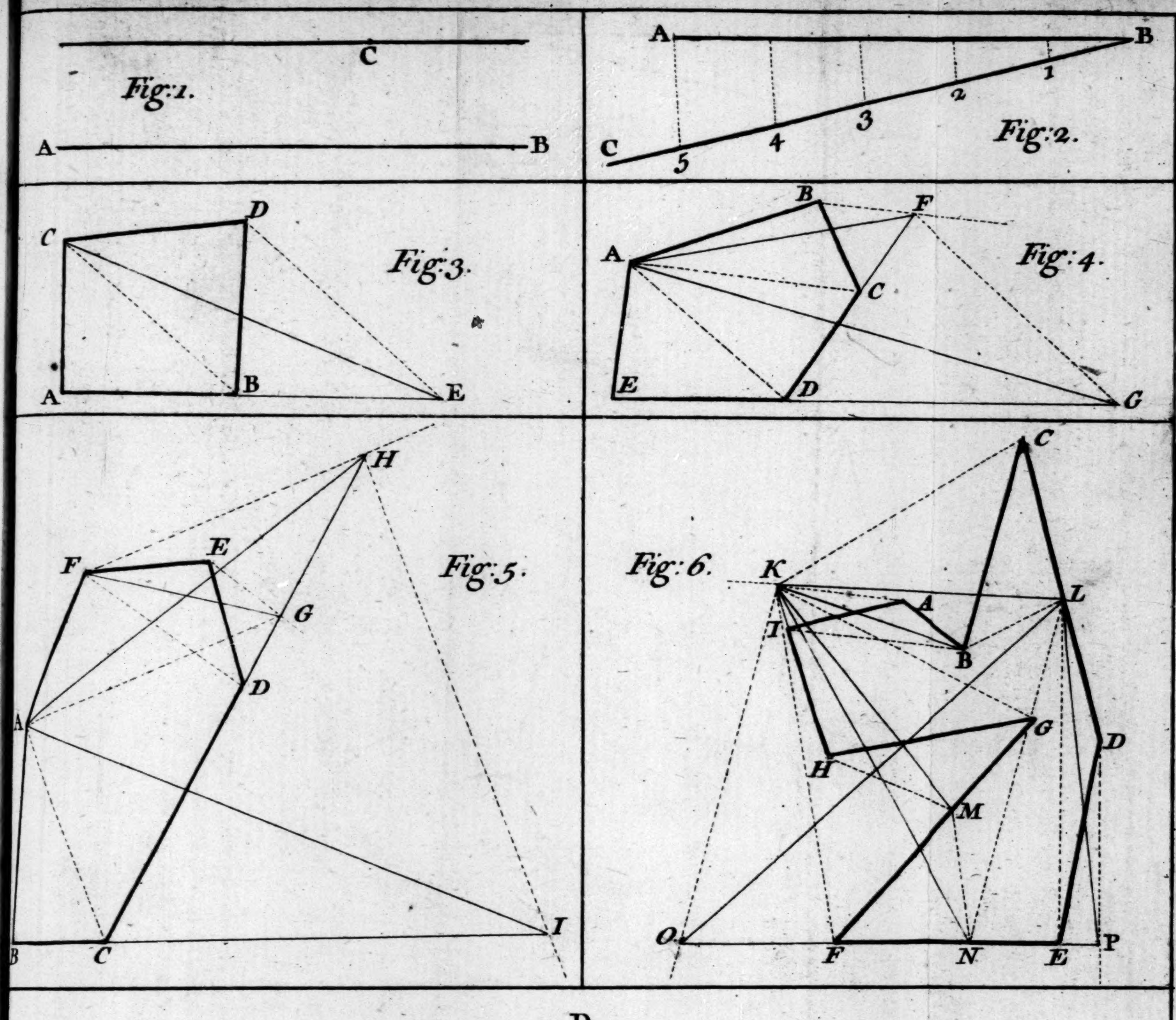


Fig: 1.
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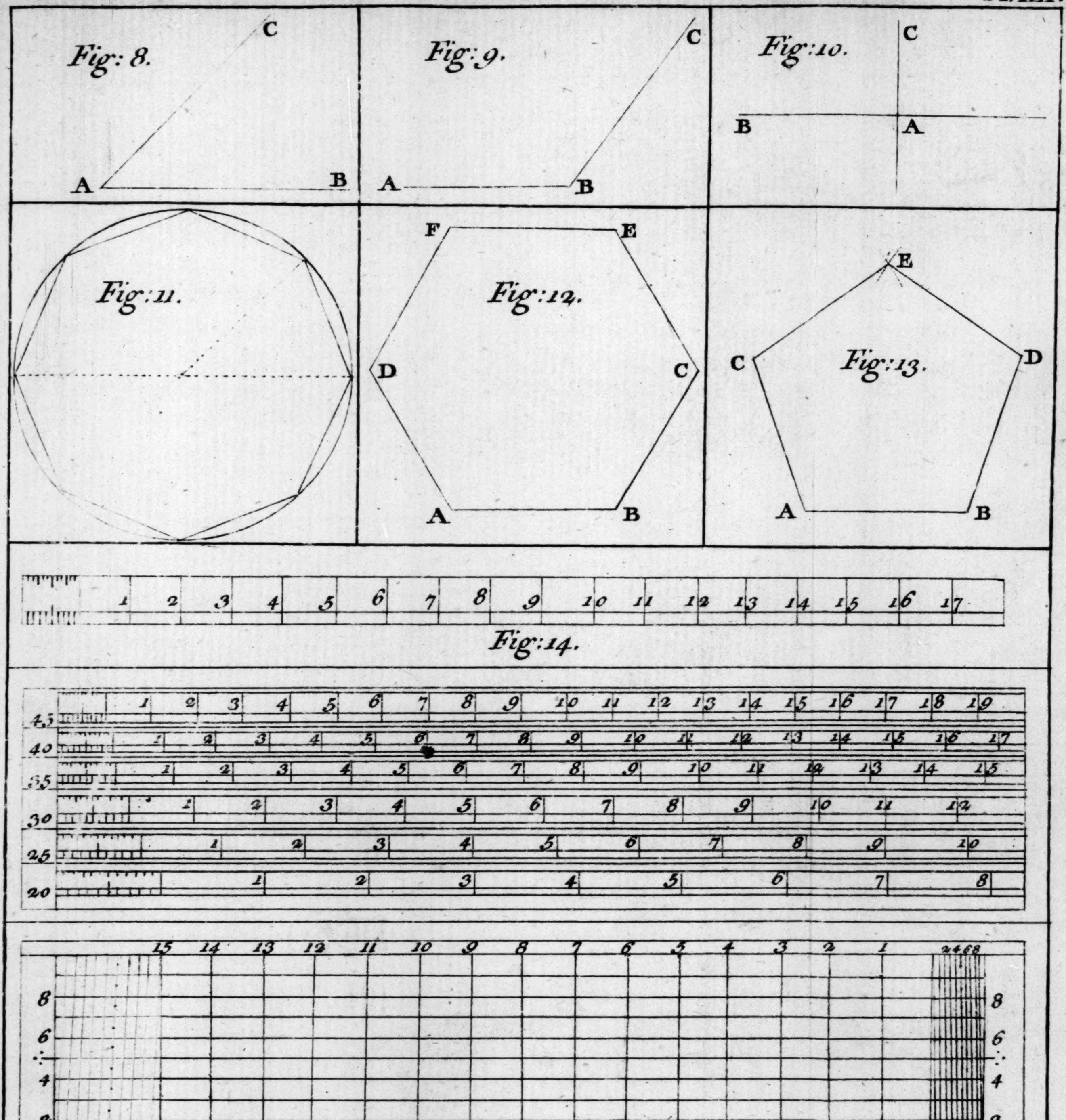


Seale Sculp.

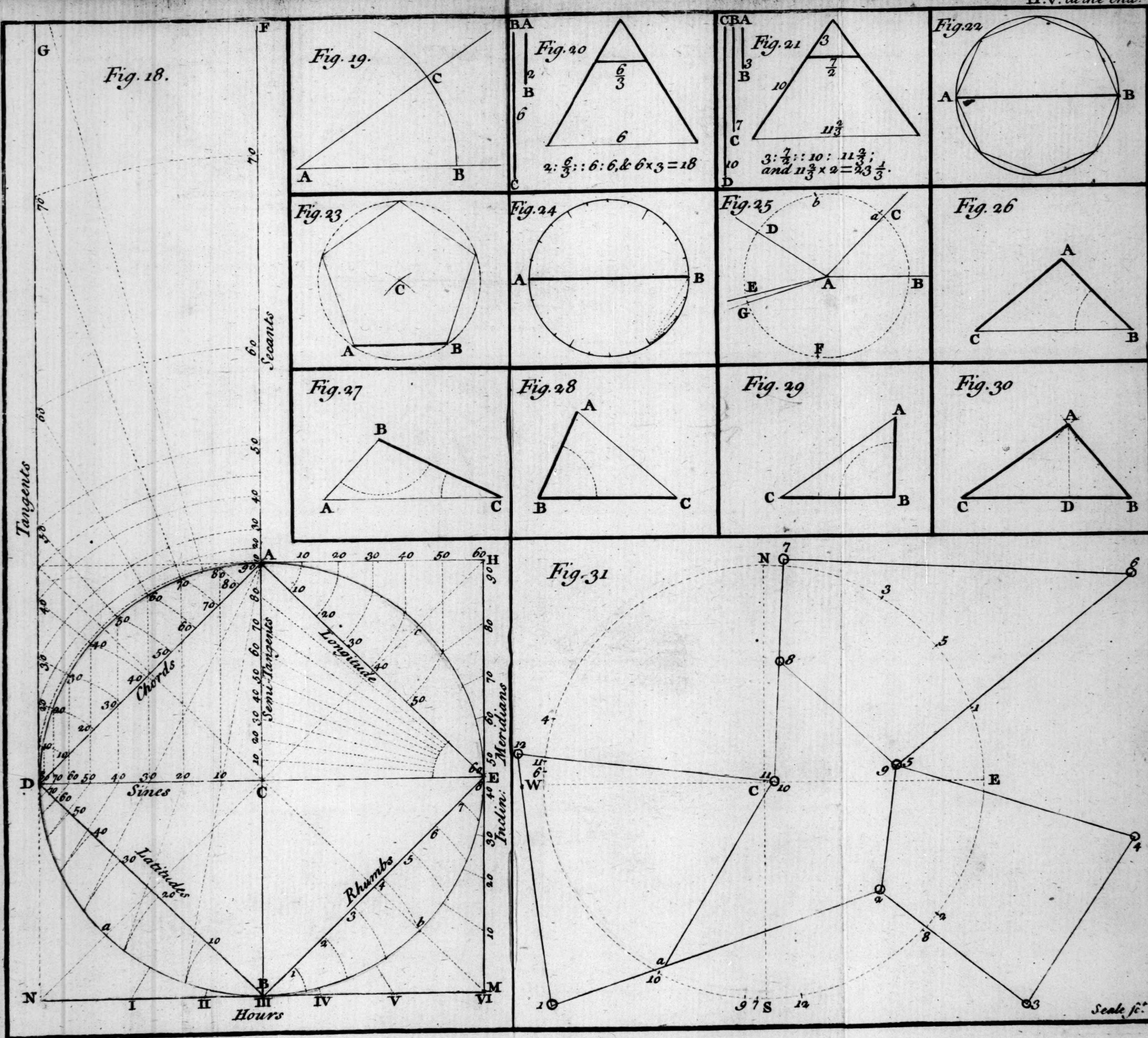
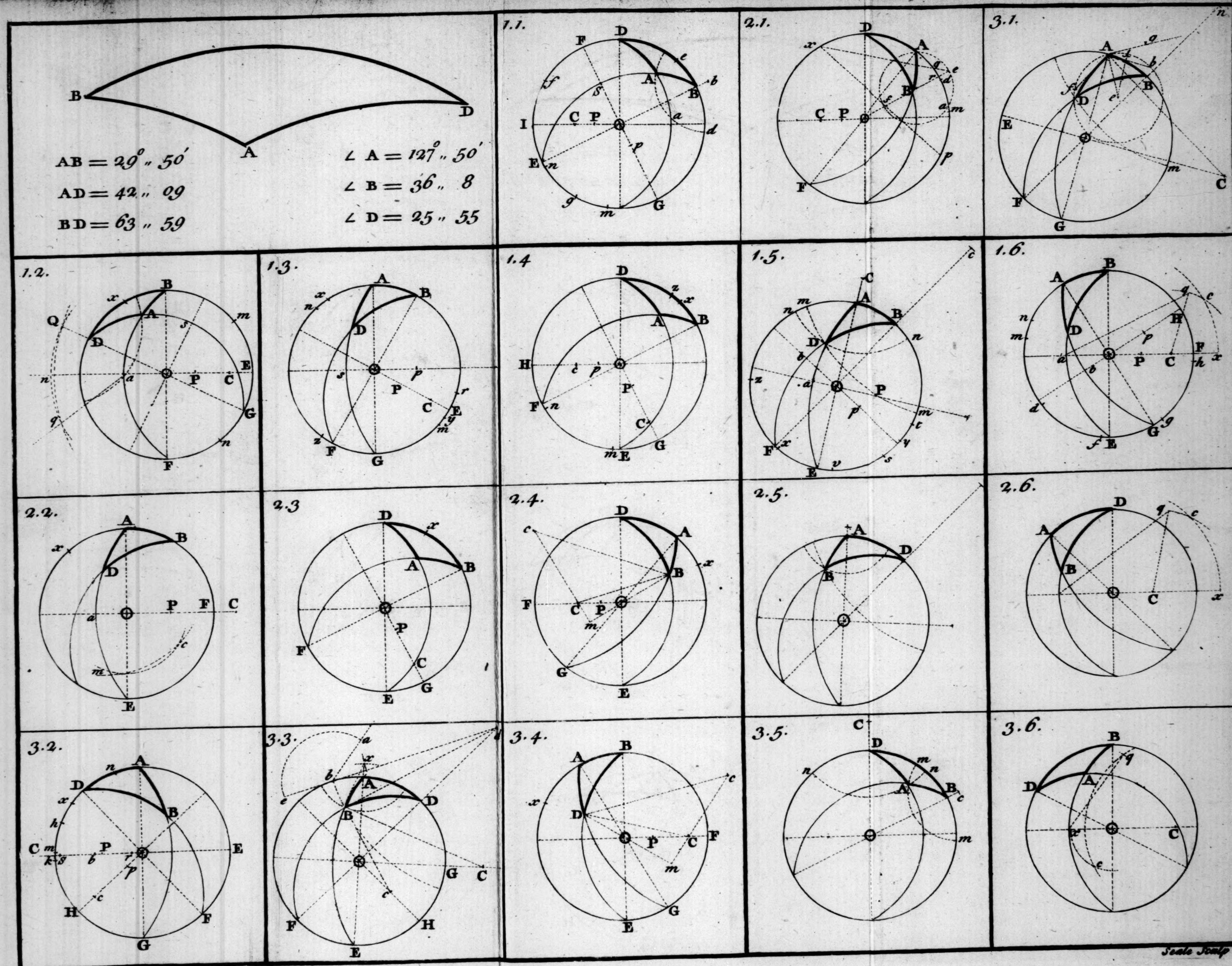


Fig. 18.
Tangents
Secants
Chords
Sines
Semi Tangents
Longitude
Meridians
Inclin:
Latitudes
Rhumbs
Hours
Fig. 19.
Fig. 20
$2 : \frac{6}{3} :: 6 : 6, \& \ 6 \times 3 = 18$
Fig. 21
$3 : \frac{7}{2} :: 10 : 11\frac{2}{3};$
and $11\frac{2}{3} \times 2 = 23\frac{1}{3}$.
Fig. 22
Fig. 23
Fig. 24
Fig. 25
Fig. 26
Fig. 27
Fig. 28
Fig. 29
Fig. 30
Fig. 31

Seale sc.


AB = 29° " 50'
AD = 42 " 09
BD = 63 " 59
∠ A = 127° " 50'
∠ B = 36 " 8
∠ D = 25 " 55
1.1.
2.1.
3.1.
1.2.
1.3.
1.4
1.5.
1.6.
2.2.
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2.4.
2.5.
2.6.
3.2.
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3.5.
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Seale Sculp